7/28/2011 SECOND HOURLY PRACTICE IV Maths 21a, O.Knill, Summer 2011

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

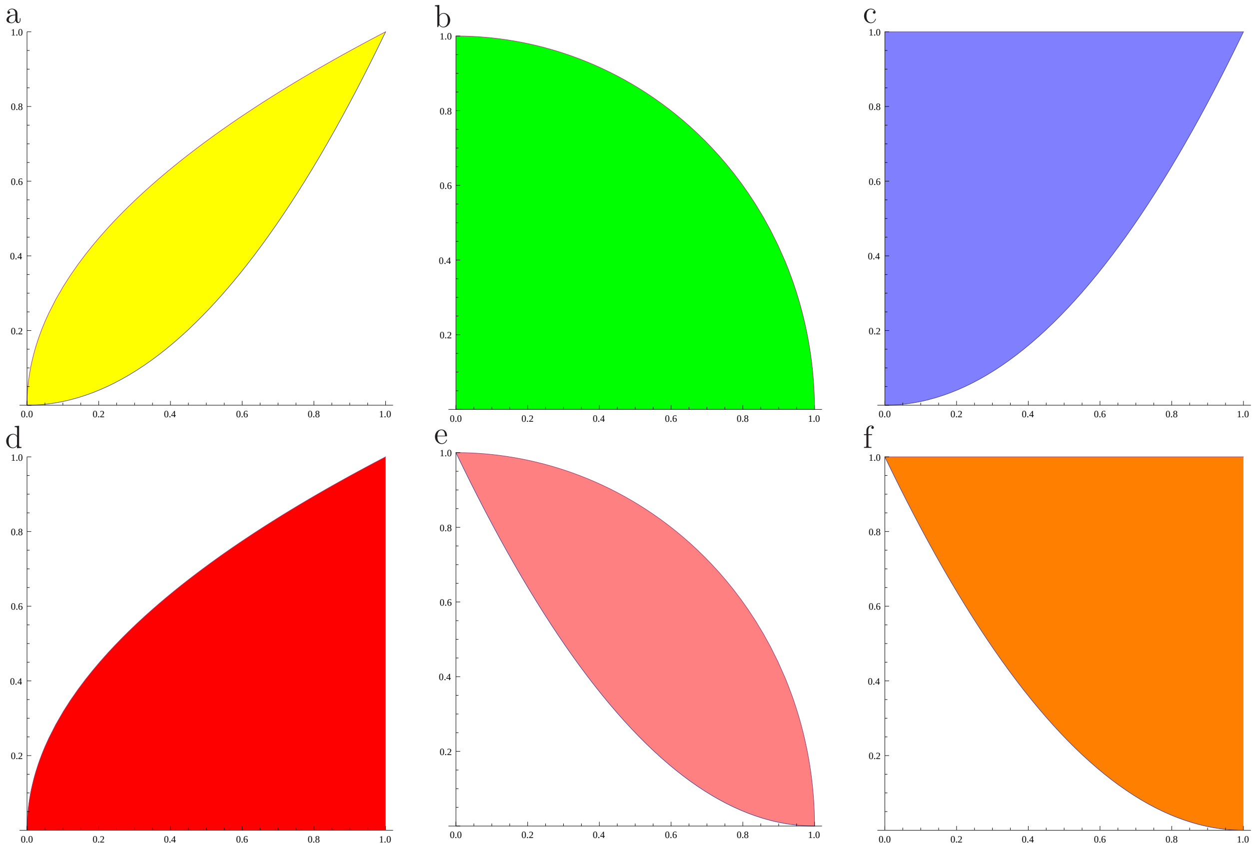
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) (20 points)

1)	T	F	$(1,1)$ is a local maximum of the function $f(x,y) = x^2y - x + \cos(y)$.
2)	T	F	If f is a smooth function of two variables, then the number of critical points of f inside the unit disc is finite.
3)	T	F	The value of the function $f(x,y) = \sin(-x+2y)$ at $(0.001, -0.002)$ can by linear approximation be estimated as -0.003.
4)	T	F	If $(1,1)$ is a critical point for the function $f(x,y)$ then $(1,1)$ is also a critical point for the function $g(x,y) = f(x^2, y^2)$.
5)	T	F	If the velocity vector $\vec{r}\,'(t)$ of the planar curve $\vec{r}(t)$ is orthogonal to the vector $\vec{r}(t)$ for all times t, then the curve is a circle.
6)	T	F	The gradient of $f(x,y)$ is normal to the level curves of f.
7)	T	F	If (x_0, y_0) is a maximum of $f(x,y)$ under the constraint $g(x,y) = g(x_0, y_0)$, then (x_0, y_0) is a maximum of $g(x,y)$ under the constraint $f(x,y) = f(x_0, y_0)$.
8)	T	F	If $\vec{u}$ is a unit vector tangent at (x,y,z) to the level surface of $f(x,y,z)$ then the directional derivative satisfies $D_u f(x,y,z) = 0$.
9)	T	F	If $\vec{r}(t) = \langle x(t), y(t) \rangle$ and $x(t), y(t)$ are polynomials, then the tangent line is defined at all points.
10)	T	F	The vector $\vec{r}_u(u,v)$ is tangent to the surface parameterized by $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$.
11)	T	F	The second derivative test allows to check whether an extremum found with the Lagrange multiplier method is a maximum.
12)	T	F	If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.
13)	T	F	Let (x_0, y_0) be a saddle point of $f(x,y)$. For any unit vector $\vec{u}$, there are points arbitrarily close to (x_0, y_0) for which ∇f is parallel to $\vec{u}$.
14)	T	F	If $f(x,y)$ has two local maxima on the plane, then f must have a local minimum on the plane.
15)	T	F	Given a unit vector v, define $g(x) = D_v f(x)$. If at a critical point, for all vectors v we have $D_v g(x) > 0$, then f is a local maximum.
16)	T	F	If $x^4 y + \sin(y) = 0$ then $y' = 4x^3/(x^4 + \cos(y))$.
17)	T	F	The critical points of $F(x,y,\lambda) = f(x,y) - \lambda g(x,y)$ are solutions to the Lagrange equations when extremizing the function $f(x,y)$ under the constraint $g(x,y) = 0$.
18)	T	F	The volume under the graph of $f(x,y) = x^2+y^2$ inside the cylinder $x^2+y^2 \leq 1$ is $\int_0^1 \int_0^{2\pi} r^3 \, d\theta dr$.
19)	T	F	The surface area of the unit sphere is 4π.
20)	T	F	The area of a disc of radius $2r$ is 4 times larger than a disc of radius r.

Problem 2) (10 points)

Match the regions with the corresponding double integrals.



Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\ dydx$
	$\int_0^1 \int_0^{\sqrt{y}} f(x,y)\ dxdy$
	$\int_0^1 \int_{y^2}^1 f(x,y)\ dxdy$

Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\ dydx$
	$\int_0^1 \int_{(1-x)^2}^1 f(x,y)\ dydx$
	$\int_0^1 \int_{(1-x)^2}^{\sqrt{1-x^2}} f(x,y)\ dydx$

Problem 3) (10 points)

a) Use the technique of linear approximation to estimate $f(\log(2)+0.001, 0.006)$ for $f(x,y) = e^{2x-y}$. (Here, log means the natural logarithm).
b) Find the equation $ax+by=d$ for the tangent line which goes through the point $(\log(2),0)$.

Problem 4) (10 points)

Find a point on the surface $g(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{8}{z} = 1$ for which the distance to the origin is a local minimum.

Problem 5) (10 points)

Find all extrema of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ on the plane and characterize them. Do you find a absolute maximum or absolute minimum amoung them?

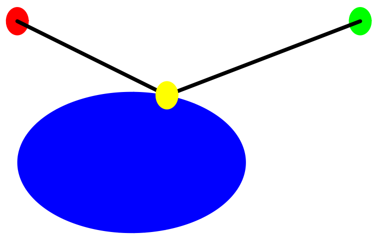
Problem 6) (10 points)

Find the surface area of the ellipse cut from the plane $z = 2x + 2y + 1$ by the cylinder $x^2 + y^2 = 1$.

Problem 7) (10 points)

Find the tangent plane to the surface $f(x, y, z) = x^3y - xy^2 + 3z = 6$ at the point $(1, 1, 2)$.

Problem 8) (10 points)



You find yourself in the desert at the point $A = (a, 1)$, completely dehydrated and almost dead. You want to reach the point $B = (b, 1)$ as fast as possible but you can not reach it without water. There is an lake inside the ellipsoid $g(x, y) = x^2 + 2y^2 = 1$. The amount of "effort" you need to go from a point (x, y) to a point (u, v) is assumed to be $(x - u)^2 + (y - v)^2$ (this is justified by the fact that if you walk for a long time, you walk less and less efficiently so that walking twice as long will take you 4 times as much effort). Find the path of least effort which connects A with $X = (x, y)$ and then with B.

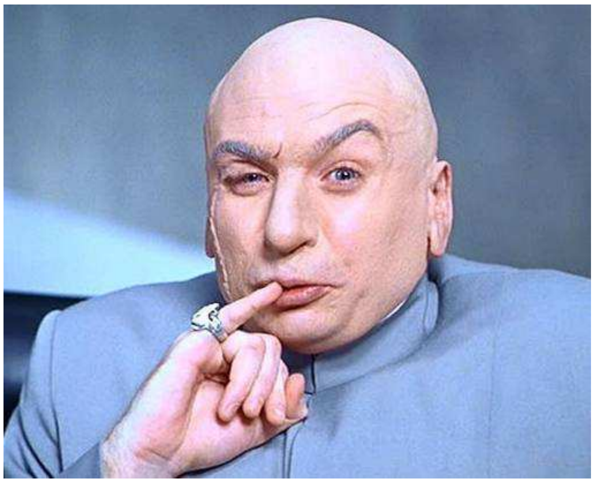
a) Which function $f(x, y)$ do you extremize? The parameters a, b are constants.
b) Write down the Lagrange equations.
c) Solve the Lagrange equations in the case $a = -1, b = 1$.

Problem 9) (10 points)

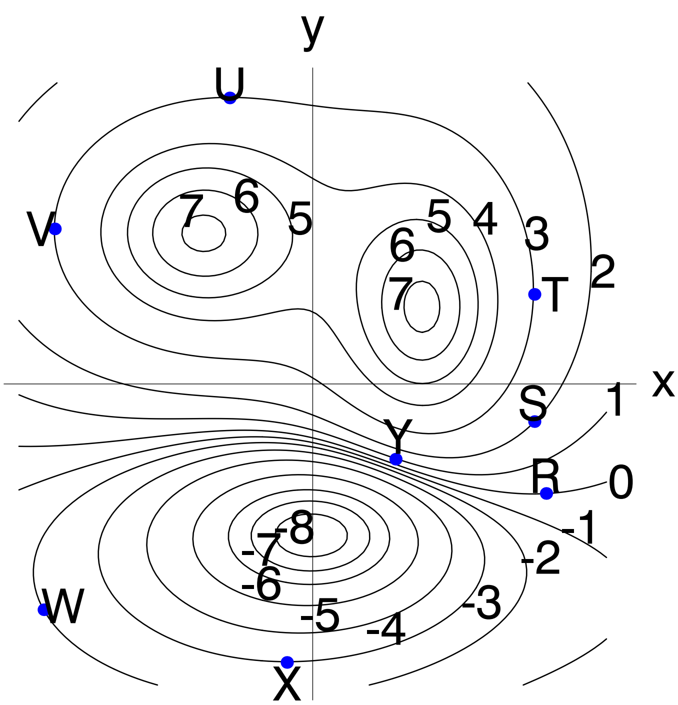
a) (5 points) Integrate $f(x,y) = x^2 - y^2$ over the unit disk $\{x^2 + y^2 \leq 1\}$.

b) (5 points) An evil integral!

$$\int_0^1 \int_0^{\sqrt{1-\theta^2}} r^2 \, dr d\theta \; .$$



Problem 10) (10 points)



a) (4 points) Circle the point at which the magnitude of the gradient vector ∇f is greatest. Mark exactly one point. Justify your answer.

R	S	T	U	V	W	X	Y

b) (3 points) Circle the points at which the partial derivative f_x is strictly positive. Mark any number of points on this question. Justify your answers.

R	S	T	U	V	W	X	Y

c) (3 points) We know that the directional derivative in the direction $(1,1)/\sqrt{2}$ is zero at one of the following points. Which one? Mark exactly one point on this question.

R	S	T	U	V	W	X	Y