7/12/2012 FIRST HOURLY PRACTICE IV Maths 21a, O.Knill, Summer 2012

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

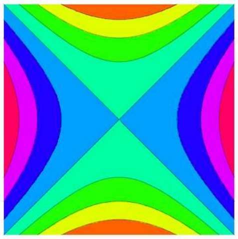
Problem 1) (20 points) No justifications are needed.

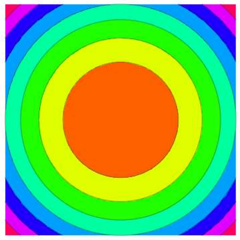
1)	T	F	The vector projection of $\langle 2,3,4 \rangle$ onto $\langle 1,0,0 \rangle$ is $\langle 2,0,0 \rangle$.
2)	T	F	The triple scalar product between three vectors is zero if and only if two of the vectors are parallel.
3)	T	F	There are two vectors $\vec{v}$ and $\vec{w}$ so that the dot product $\vec{v} \cdot \vec{w}$ is equal to the length of the cross product $|\vec{v} \times \vec{w}|$.
4)	T	F	The distance between two spheres of radius 1 whose centers have distance 10 is 8.
5)	T	F	If two vectors $\vec{v}$ and $\vec{w}$ are both parallel and perpendicular, then one of the vectors must be the zero vector.
6)	T	F	The curvature $\kappa(\vec{r}(t))$ is always smaller or equal than the length $|\vec{r}\,''(t)|$ of the acceleration vector $\vec{r}\,''(t)$.
7)	T	F	The curve $\vec{r}(t) = \langle \cos(t)\sin(t), \sin(t)\sin(t), \cos(t) \rangle$ is located on a sphere.
8)	T	F	The surface $x^2 + y^2 + z^2 = 2z$ is a sphere.
9)	T	F	The length of the vector $\langle 4,2,4 \rangle$ is an integer.
10)	T	F	The curvature of the curve $\langle 2\cos(t^3), 2\sin(t^3), 1 \rangle$ is constant 2.
11)	T	F	The graph of the function $f(x,y) = x^2 - y^2$ is called an elliptic paraboloid.
12)	T	F	The equation $\phi = 3\pi/2$ in spherical coordinates defines a plane.
13)	T	F	The vector $\langle 1,2,3 \rangle$ is perpendicular to the plane $2x + 4y + 6z = 4$.
14)	T	F	The cross product between $\langle 2,3,1 \rangle$ and $\langle 1,1,1 \rangle$ is 6.
15)	T	F	The curve $\vec{r}(t) = \langle \cos(t), t^2, \sin(t) \rangle, 1 \leq t \leq 9$ and the curve $\vec{r}(t) = \langle \cos(t^2), t^4, \sin(t^2) \rangle, 1 \leq t \leq 3$ have the same length.
16)	T	F	If a stone falls for 3 seconds from height $z = h$ to the ground $z = 0$ with gravitational acceleration -10 then the height is 30 meters.
17)	T	F	The point $(1,-1,1)$ has the spherical coordinates the form $(\rho, \theta, \phi) = (\sqrt{3}, \pi/4, \pi/4)$.
18)	T	F	The point $(1,-1,1)$ has the cylindrical coordinates the form $(r, \theta, z) = (\sqrt{3}, \pi/4, 1)$.
19)	T	F	The distance between two parallel lines in space is the distance of a point on one line to the other line.
20)	T	F	For two nonzero arbitrary vectors $\vec{v}$ and $\vec{w}$ the identity $\mathrm{Proj}_{\vec{v}}(\vec{v} \times \vec{w}) = \vec{0}$ holds.

Total

Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match contour maps with functions $f(x,y)$. Enter O, where no match.


I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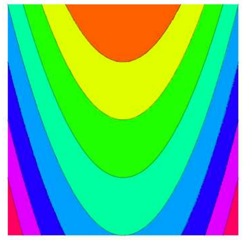
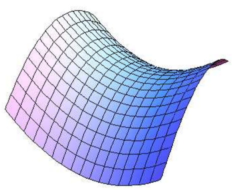
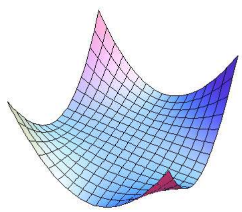
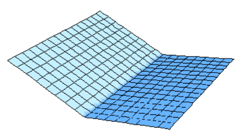

III

Function $f(x,y) =$	Enter I,II,III
x^2+y^2	
x^2-y^2	
x^2-y	
$x-y^2$	

b) (3 points) Match the graphs with the functions $f(x,y)$. Enter O, where no match.

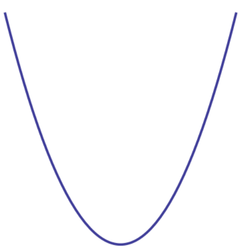

I

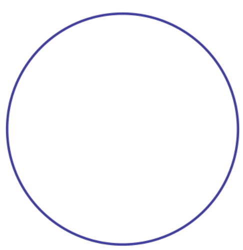

II


III

Function $f(x,y) =$	Enter I,II,III
$x-y$	
$\|x\|-y$	
x^2-y^2	
x^2y^2	
$x-y^3$	

c) (2 points) Match the curves with their parametrizations $\vec{r}(t)$. Enter O, where no match.


I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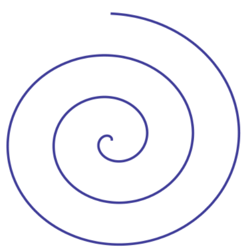
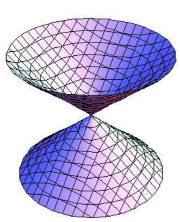
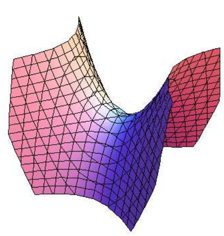

III

Curve $\vec{r}(t) =$	Enter I,II,III
$\langle t, t^2\rangle$	
$\langle t^4, 1+2t^4\rangle$	
$\langle -t\sin(t), t\cos(t)\rangle$	
$\langle \sin(t), \cos(t)\rangle$	

d) (3 points) Match level surfaces with definition $g(x,y,z)=0$. Enter O, where no match.


I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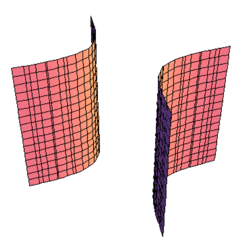
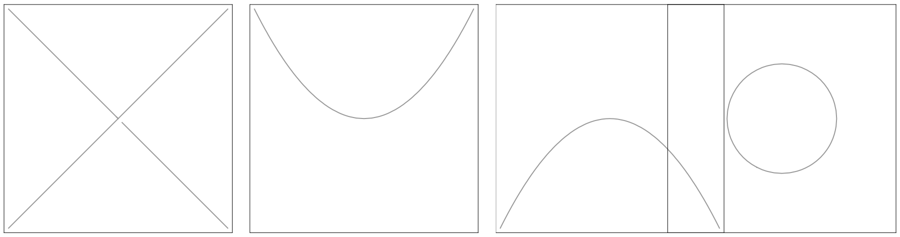

III

Function $g(x,y,z) =$	Enter I,II,III
$x^2+y^2-z^2$	
x^2-y^2-1	
x^2-y^2-z	
x^2+y^2-z	
$x-y+z$	

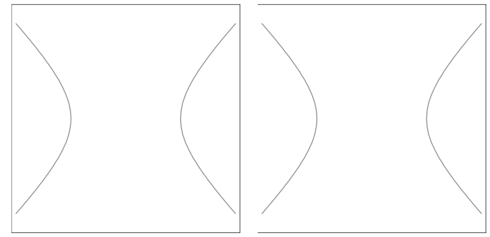
Problem 3) (10 points) No justifications are needed in this problem

3a) (5 points) Matching traces with surfaces.

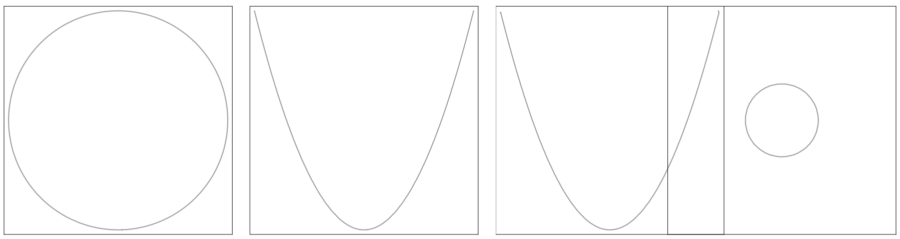
xy-trace yz-trace xz-trace xy-trace yz-trace xz-trace



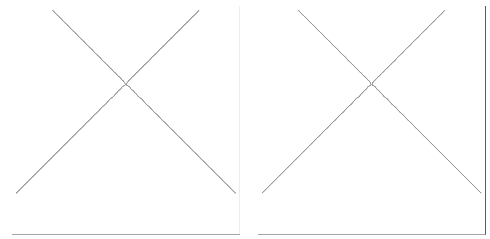
A



B



C

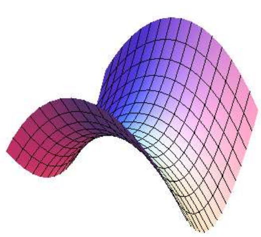


D

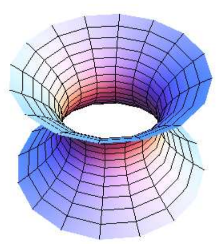
The figures above show the xy-trace, (the intersection of the surface with the xy-plane), the yz-trace (the intersection of the surface with the yz-plane), and the xz-trace (the intersection of the surface with the xz-plane). Match the following equations with the traces. No justifications required.

Enter A,B,C,D,E,F here	Equation
	$x^2 + y^2 - (z - 1/3)^2 = 0$
	$x^2 - y^2 + z = 0$
	$x^2 + y^2 - z^2 - 1 = 0$
	$x^2 + y^2 - z = 1$

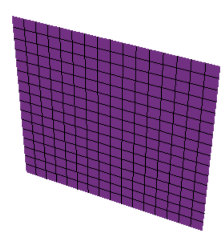
3b) (5 points) Matching parametrized surfaces.



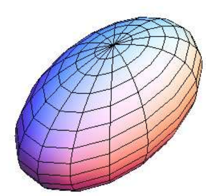
I



II



III



IV

Match the parametric surfaces with their parameterization. No justifications are needed.

Enter I,II,III,IV here	Parametrization
	$\vec{r}(u,v) = \langle u, v, v^2 - u^2 \rangle$
	$\vec{r}(u,v) = \langle \cos(u)\sin(v), 2\sin(u)\sin(v), \cos(v) \rangle$
	$\vec{r}(u,v) = \langle (v^2+1)\cos(u), (v^2+1)\sin(u), v \rangle$
	$\vec{r}(u,v) = \langle u, 3, v \rangle$

Problem 4) (10 points)

We want to find the distance between the lines $x = y = z$ and $(x-1)/2 = (y-2)/3 = (z-4)/4$.

a) (4 points) Find a parametrization for each of the two lines.

b) (6 points) Find the distance between the two lines.

Problem 5) (10 points)

At the **independence day** celebration on July 4, 2010 in Boston, two rockets were launched at the same time. Their paths follow parabola:

$$\vec{r}(t) = \langle t, t, 5 - t^2 \rangle ,$$

$$\vec{R}(t) = \langle 2 - t, t, 4 + t - t^2 \rangle .$$

a) (3 points) They collide at some time $t = t_0$. Find this time.

b) (4 points) Compute the two velocity vectors $\vec{r}\,'(t)$ and $\vec{R}\,'(t)$ at $t = t_0$.

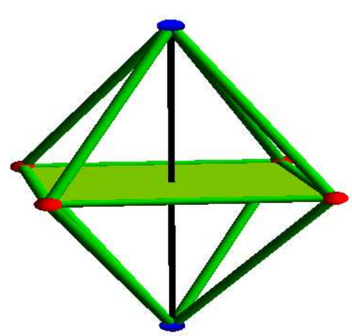
c) (3 points) Determine the cos of the angle of intersection between the curves at the impact point.

Problem 6) (10 points)

An octahedron has 4 vertices $A = (-1, -1, 0), B = (1, -1, 0), C = (1, 1, 0), D = (-1, 1, 0)$ in the xy plane. Two other vertices are at $E = (0, 0, a)$ and $F = (0, 0, -a)$.

a) (4 points) For which positive value of a is the distance between A and F equal to 2 and the solid a regular octahedron?

b) (6 points) Find the distance between A and the line connecting the points B and F.



Problem 7) (10 points)

Let $\vec{v} = \langle 3, 4, 5 \rangle$, $\vec{w} = \langle 1, 1, 1 \rangle$. Compute the following expressions:

a) (2 points) the area of the parallelogram spanned by $\vec{v}$ and $\vec{w}$,

b) (2 points) the vector $(\vec{v} \times \vec{w}) \times \vec{w}$,

c) (2 points) the scalar $\vec{v} \cdot \vec{w}$,

d) (2 points) the vector $\text{Proj}_{\vec{v}}(\vec{w})$,

e) (2 points) $\cos(\alpha)$, where α is the angle between $\vec{v}$ and $\vec{w}$.

Problem 8) (10 points)

a) (7 points) Find the arc length of the curve

$$\vec{r}(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle$$

from $0 \leq t \leq 3$.

b) (3 points) Find the unit tangent vector $\vec{T}(t)$ to $\vec{r}(t)$ at time $t = \sqrt{\pi/2}$.

Problem 9) (10 points)

A stunt man wants to jump from the golden gate bridge from 80 meters starting at $\vec{r}(0) = \langle 0, 0, 80 \rangle$. He moves from the platform with initial velocity $\vec{r}\,'(0) = \langle 0, 1, 0 \rangle$. By the way, the swiss **Oliver Favre** holds the record of jumping from 54 meters.

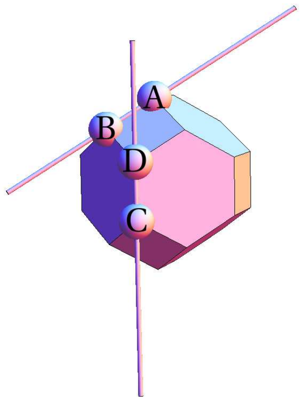
a) (2 points) How long does the diver fall, if the acceleration is $\langle 0, 0, -10 \rangle$.

b) (3 points) Find the trajectory $\vec{r}(t)$ of the stunt man.

c) (3 points) At which point does he hit the water surface $z = 0$?

d) (2 points) With what speed does he hit the surface?

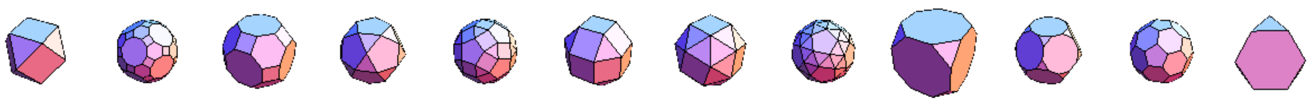
Problem 10) (10 points)



A **truncated octahedron** has an edge connecting the vertices $A = (-1, 3, 0), B = (-1, 1, -1)$ and an edge connecting the vertices $C = (-3, -1, 0), D = (-3, 1, 0)$.

a) (5 points) Find the distance of C to the line through A, B.

b) (5 points) Find the distance between the line L through A, B and the line K through C, D.



Here are the remaining 12 **Archimedean solids**. These are polyhedra bound by different types of regular polygons but for which each vertex of the polyhedron looks the same. There are 13 such semiregular polyhedra. Archimedes studied them first in 287BC. Kepler was the first to describe the complete set of 13 in his work **"Harmonices Mundi"** of 1619.