7/12/2012 FIRST HOURLY PRACTICE V **Maths 21a, O.Knill, Summer 2012**

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) (20 points) No justifications are needed.

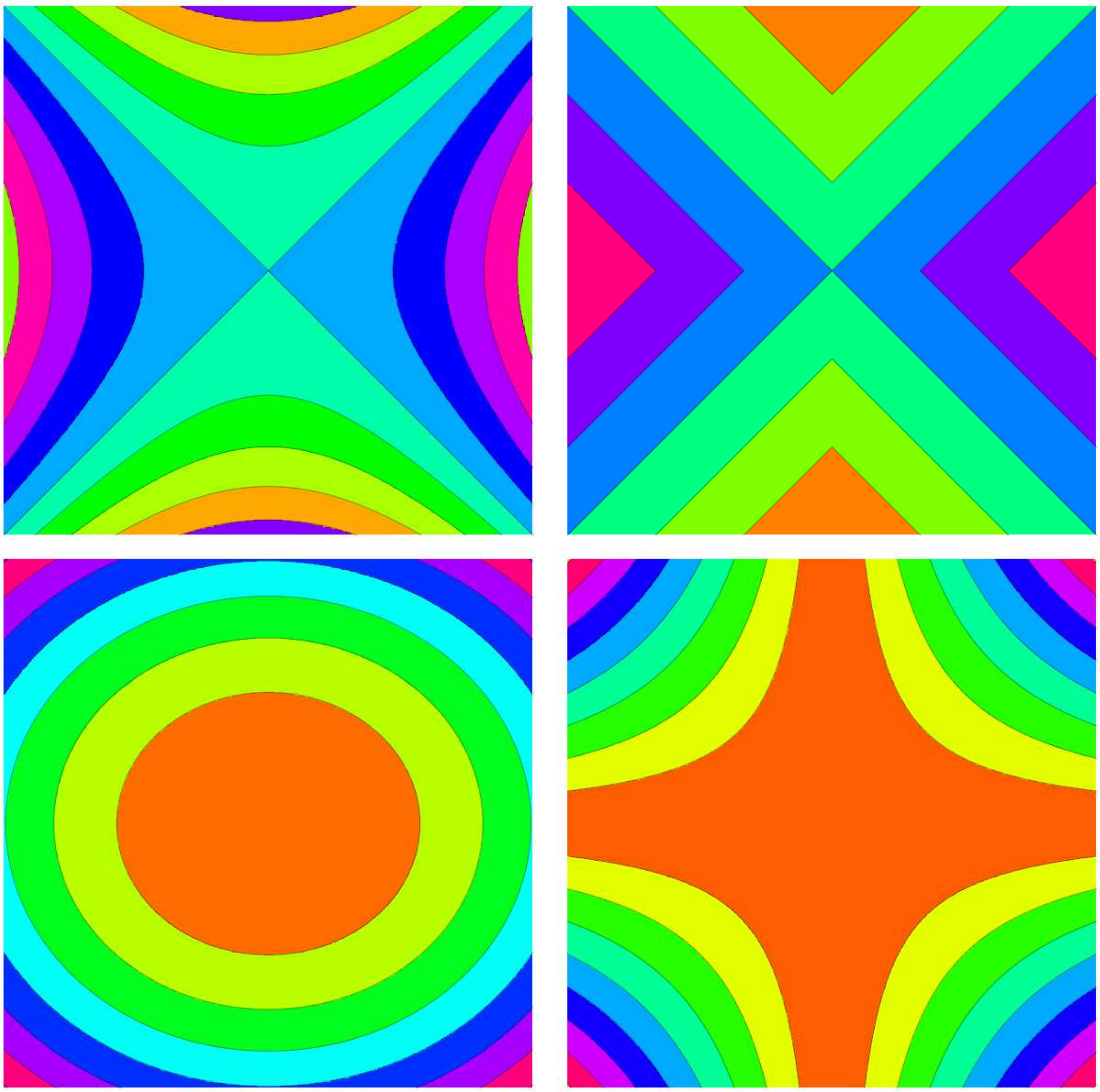
1)	T	F	The vector $\vec{v} = \langle 1,3,5 \rangle$ is perpendicular to the plane $x + 3y + 5z = 1$.
2)	T	F	The set of points which satisfy $x^2 - y^2 + z^2 - 2z + 1 = 0$ forms a double cone.
3)	T	F	The set of points in $\mathbf{R}^3$ which have distance 1 from a point form a cylinder.
4)	T	F	The surface $-x^2 + y^2 + z^2 = 1$ is called a one-sheeted hyperboloid.
5)	T	F	The two vectors $\langle 2,3,0 \rangle$ and $\langle 6,-4,5 \rangle$ are orthogonal to each other.
6)	T	F	Two nonzero vectors are parallel if and only if their dot product is 0.
7)	T	F	The cross product is associative: $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \times \vec{w}$.
8)	T	F	Every vector contained in the line $\vec{r}(t) = \langle 1+2t, 1+3t, 1+4t \rangle$ is parallel to the vector $(2,3,4)$.
9)	T	F	The line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$ hits the plane $2x + 3y + 4z = 9$ at a right angle.
10)	T	F	Two planes $ax + by + cz = d$ and $ux + vy + wz = e$ intersect in a line if $\lvert\langle a,b,c \rangle \times \langle u,v,w \rangle\rvert > 0$.
11)	T	F	The equations $x - 2 = y - 3 = z - 4$ describe a line which contains the vector $\langle 1,1,1 \rangle$.
12)	T	F	In spherical coordinates, the equation $\cos(\phi) = \sin(\phi)$ defines a cone.
13)	T	F	A point with spherical coordinates $(\rho, \theta, \phi) = (1, \pi/2, \pi/4)$ has cylinder coordinates $(r, \theta, z) = (1/\sqrt{2}, \pi/2, 1/\sqrt{2})$.
14)	T	F	If in rectangular coordinates, a point is given by $(1,0,-1)$, then its spherical coordinates are $(\rho, \theta, \phi) = (\sqrt{2}, \pi/2, 3\pi/4)$.
15)	T	F	The volume of a parallelepiped spanned by $(1,0,0), (0,1,0)$ and $(0,1,1)$ is equal to 2.
16)	T	F	The vector projection of the vector $(2,4,5)$ onto the vector $(1,1,0)$ is $(3/2, 3/2, 0)$.
17)	T	F	If $g(x,y,z) = 0$ is a surface given implicitly, then $\vec{r}(u,v) = \langle u, v, g(u,v,g(u,v,1)) \rangle$ is a parametrization of the surface.
18)	T	F	If $z = g(x,y)$ is a graph then $\vec{r}(u,v) = \langle u, v, g(u,v) \rangle$ is a parameterization of the surface.
19)	T	F	The distance from the point $P = (1,1,1)$ to the x axes is $\sqrt{2}$.
20)	T	F	The distance between the point $P = (1,1,1)$ and the xy plane is $\sqrt{2}$.

Total

Problem 2) (10 points)

a) (3 points) Match the contour maps with the corresponding functions $f(x, y)$ of two variables. Enter O if no figure matches. No justifications are needed.

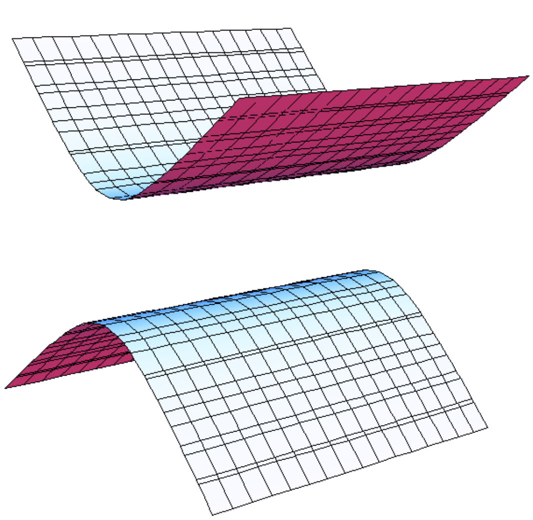
I II III IV

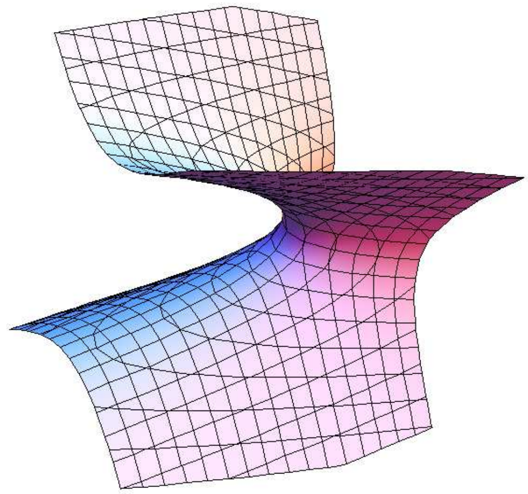


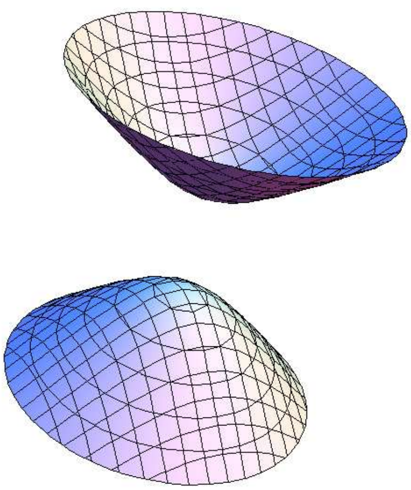
Enter I,II,III,IV or O	Function $f(x, y)$	Enter I,II,III,IV or O	Function $f(x, y)$
	$f(x, y) = x\cos(y)$		$f(x, y) = x^2 - y^2$
	$f(x, y) = 3x^2 + 4y^2$		$f(x, y) = \|xy\|$
	$f(x, y) = \cos(x)$		$f(x, y) = \|x\| - \|y\|$

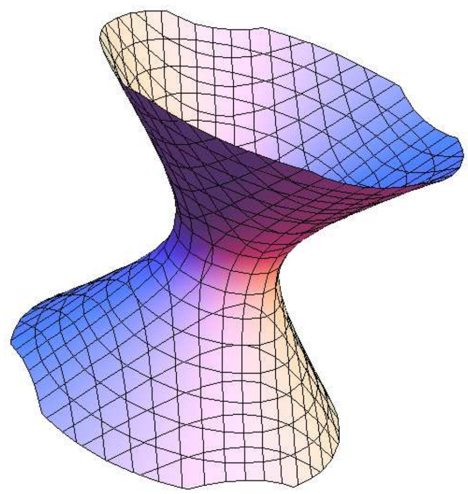
b) (4 points) Match the quadrics with the functions. No justifications are necessary.

a b c d







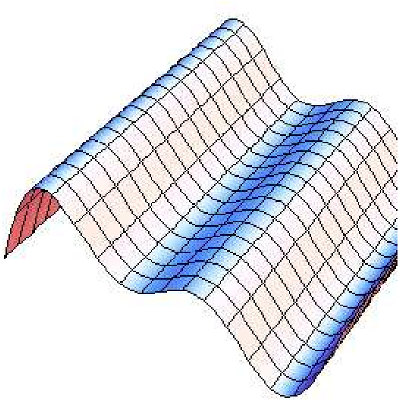


Enter a,b,c,d here	Equation
	$x + y^2 - z^2 - 1 = 0$
	$y^2 - z^2 + 1 = 0$

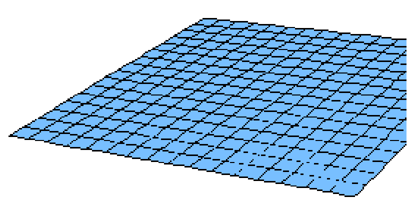
Enter a,b,c,d here	Equation
	$x^2 + y^2 - z^2 + 1 = 0$
	$x^2 + y^2 - z^2 - 1 = 0$

c) (3 points) Match the equation with their graphs. No justifications are necessary.

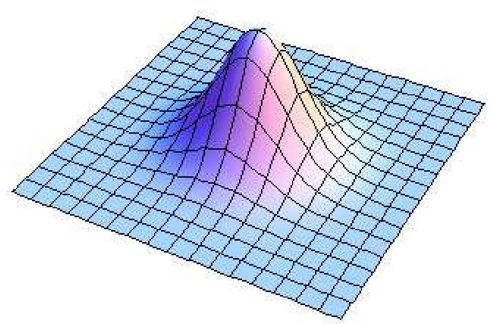
A



B



C



Enter A,B,C here	Equation	Enter A,B,C here	Equation	Enter A,B,C here	Equation
	$z = e^{-x^2-y^2}$		$z = x\cos(x)$		$z = x - y$

Problem 3) (10 points)

a) (5 points) Surfaces $z = f(x,y)$ which are graphs can be written implicitly as $g(x,y,z) = 0$, parametrized as $\vec{r}(u,v)$. For example, $z = \log(xy)$ is given by $g(x,y,z) = 0$ with $g(x,y,z) = z - \log(xy)$ or parametrized as $\vec{r}(u,v) = \langle u, v, \log(uv) \rangle$. Complete the following table by filling in the choices $A - J$ below. No justifications are needed in this problem.

$z = f(x,y)$ for	$g(x,y,z) = 0$	$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$
		$\langle v\cos(u), v\sin(u), v \rangle$
	$x + y - 2z = 0$	
$f(x,y) = x^2 - y^2$		
		$\langle \cos(u)\sin(v), \sin(u)\sin(v), \cos(v) \rangle, v < \pi/2$
	$z - \sin(xy) = 0$	
$f(x,y) = x$		

A) $f(x,y) = x - y$
B) $f(x,y) = x^2 + y^2$
C) $z - x^2 - y^2$
D) $\langle 1 + u + v, 1 + u - v, u \rangle$
E) $z - x^2 + y^2$.
F) $f(x,y) = \sqrt{1 - x^2 - y^2}$
G) $\langle u, v, u^2 - v^2 \rangle$
H) $x^2 + y^2 + z^2 - 1 = 0$
I) $\langle u, v, u \rangle$
J) $z - x = 0$

b) (5 points)

Quantity	Check if it depends on parametrization of $\vec{r}$	Is a vector
Curvature of $\vec{r}(t)$		
Arc length of $\vec{r}(t)$ from 0 to 1		
Acceleration of $\vec{r}(t)$		
Jerk of $\vec{r}(t)$		
Speed of $\vec{r}(t)$		
Unit tangent of $\vec{r}(t)$		
Normal of $\vec{r}(t)$		
Binormal of $\vec{r}(t)$		
$\nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$		
$\vec{r}\,'(t) \times \vec{r}\,''(t)$		

Problem 4) (10 points)

A billard ball starts at $A = (1,1,0)$, travels along the vector $\vec{u} = \langle 2, -2, 0 \rangle$ to other point B where it bounces off an other ball. It travels from there along the vector $\vec{v} = \langle -3, 4, 0 \rangle$ to a third point C, where it bounces off a wall, rolling along the vector $\vec{w} = \langle 1, 1, 0 \rangle$ to its final destination D. In other words, you know A, $\vec{AB} = \vec{u}$, $\vec{BC} = \vec{v}$ and $\vec{CD} = \vec{w}$.

a) (5 points) What are the coordinates of the point D?

b) (5 points) Find the total distance traveled by the ball along the path A, B, C, D.

Problem 5) (10 points)

a) (5 points) Find the symmetric equation of the line which contains the point $P = (3, 4, 1)$ and the point $Q = (5, 5, 5)$.

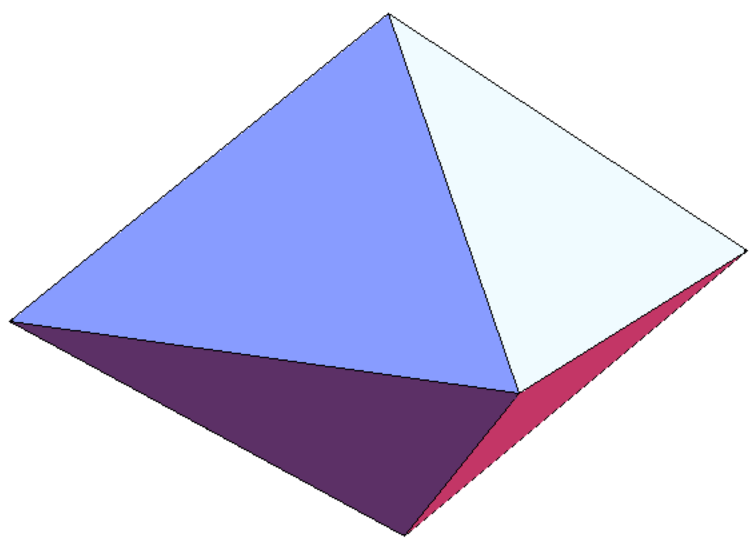
b) (5 points) What is the equation of the plane perpendicular to the line in a) which passes through the point $P = (3, 4, 1)$?

Problem 6) (10 points)

We look at a polyhedron which has the shape of a scaled octahedron. Its vertices are $A = (1, 1, 0), B = (-1, 1, 0), C = (-1, -1, 0), D = (1, -1, 0), E = (0, 0, 1), F = (0, 0, -1)$.

a) (5 points) Parametrize the line L passing through A,E and the line K passing through B,F.

b) (5 points) Find the distance between these two lines L and K.

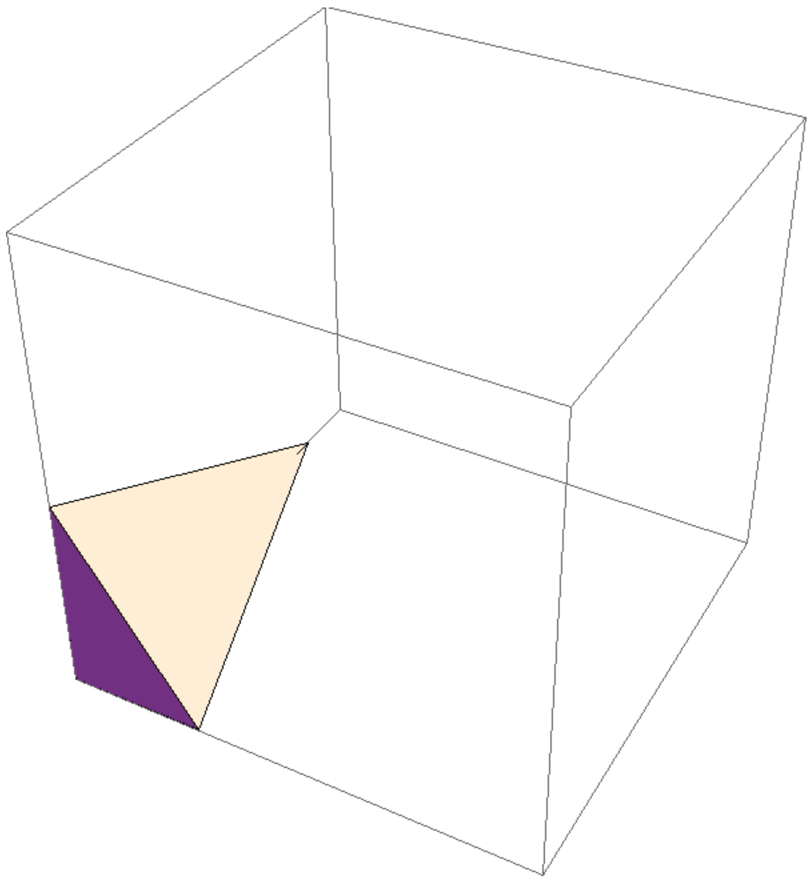


Problem 7) (10 points)

The plane $3x + y + 2z = 6$ cuts out a triangle T from the octant $x > 0, y > 0, z > 0$. This triangle as well as the coordinate planes bound a solid E in space.

a) (5 points) Find the area of this triangle $T = ABC$.

b) (5 points) The volume of the solid E is known to be 1/6'th of the volume of the parallelepiped spanned by $\vec{OA}, \vec{OB}, \vec{OC}$. Find the volume of E.



Problem 8) (10 points)

a) (2 points) Find the dot product of $\langle 3, 4, 5 \rangle$ and $\langle 3, 2, 1 \rangle$.

b) (2 points) Find the vector projection of the vector $\langle 4, 5, 6 \rangle$ onto the vector $\langle 3, 3, 3 \rangle$.

c) (2 points) Do the vectors $\langle 1, -1, -1 \rangle$ and $\langle 4, -5, 6 \rangle$ form an acute or obtuse angle?

d) (2 points) What is the triple scalar product of $\vec{i} + \vec{j}, \vec{j}$ and $\vec{i} - \vec{j}$?

e) (2 points) Find the cross product of $\langle 1, 1, 2 \rangle$ and $\langle 3, 4, 5 \rangle$.

Problem 9) (10 points)

a) (3 points) Parametrize the plane containing the three points $A = (1, 1, 1)$, $B = (1, 3, 2)$ and $C = (3, 4, 5)$.

b) (3 points) Parametrize the sphere which is centered at $(1, 1, 1)$ and has radius 3.

c) (4 points) Parametrize the set of points which have distance 2 from the x-axes.

Problem 10) (10 points)

An apple at position $(0, 0, 20)$ rests 20 meters above Newton's head, the tip of whose nose is at $(1, 0, 0)$. The apple falls with constant acceleration $\vec{r}\,''(t) = \langle a, 0, -10\rangle$ (where $\langle 0, 0, -10\rangle$ is caused by gravity and $\langle a, 0, 0\rangle$ by the wind) precisely onto the nose of Newton. Find the wind force $\langle a, 0, 0\rangle$ which achieves this. Give a parametrization for the path along which the apple falls.