7/25/2013 SECOND HOURLY PRACTICE III Maths 21a, O.Knill, Summer 2013

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Provide details to all computations except for problems 1-3.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) True/False questions (20 points), no justifications needed

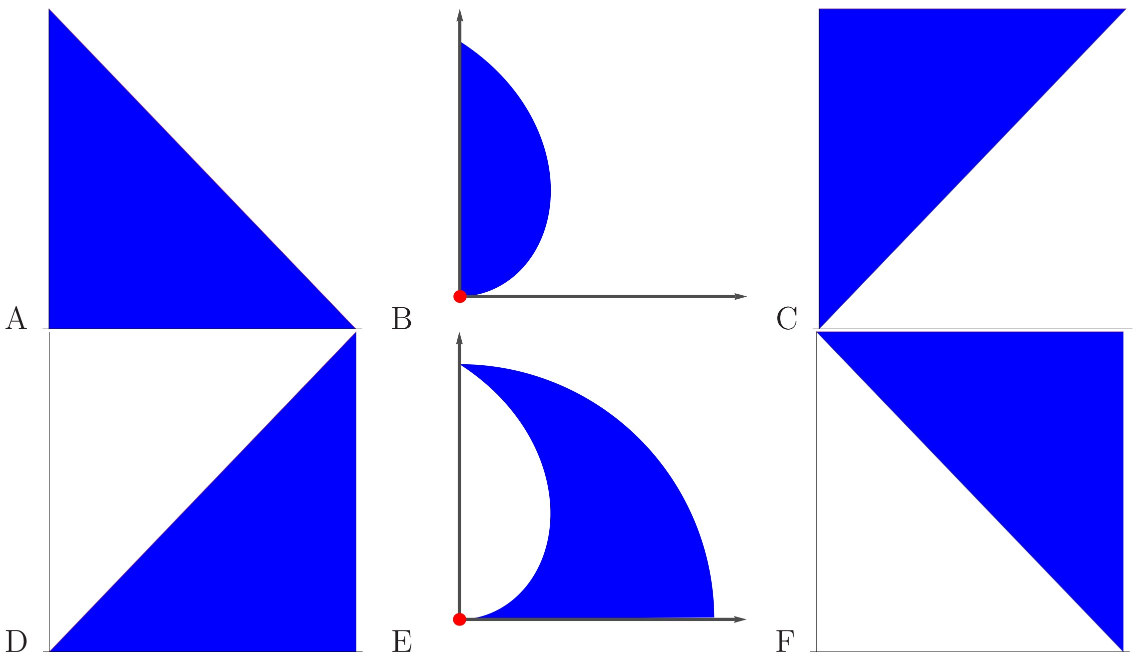
Mark for each of the 20 questions the correct letter. No justifications are needed.

1)	T	F	The partial differential equation $u_t = u_{xx}$ is called the heat equation.
2)	T	F	Every function $f(x,y)$ in 2 variables has either a local maximum or a local minimum or a saddle point.
3)	T	F	If $\iint_R f(x,y)\ dxdy = 0$, then the function $f(x,y)$ is everywhere zero on $R = \{x^2 + y^2 \leq 1\ \}$.
4)	T	F	If f has a local maximum at $(1,0)$ and $g(x,y) = x^2 + y^2 = 1$ is a constraint then the Lagrange equations $\nabla f = \lambda \nabla g, g = 1$ are satisfied.
5)	T	F	The directional derivative is always smaller than the length of the gradient.
6)	T	F	The linearization of a function $f(x,y)$ at $(0,0)$ is a function of the form $L(x,y) = ax + by + c$.
7)	T	F	The surface area of a surface $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$ defined on $(u,v) \in R$ is smaller or equal to than $\int\int_R |\vec{r}_u| \cdot |\vec{r}_v|\ dudv$.
8)	T	F	A rectangle is both type I and type II.
9)	T	F	If the function $D(x,y) = f_{xx}f_{yy} - f_{xy}^2$ is negative everywhere, then every critical point of f is a saddle point.
10)	T	F	If two functions f and g have the same critical points, then $f = \lambda g$.
11)	T	F	If (x,y) is not a critical point, then the directional derivative $D_{\vec{v}}f(x,y)$ can take both positive and negative values for different choices of $\vec{v}$.
12)	T	F	Using linearization of $f(x,y) = x/y$ we can estimate $1.01/1.1 = f(1.01, 1.1) \sim 1 + 0.01 - 0.1 = 0.91$.
13)	T	F	If $(1,1)$ is a local maximum, then $D = f_{xx}^2 f_{yy}^2 - f_{xy}^2 \geq 0$.
14)	T	F	$\int_0^2 \int_0^3 f(x,y)\ dxdy = \int_0^2 \int_0^3 f(x,y)\ dydx$ for any continuous function $f(x,y)$.
15)	T	F	If $\vec{r}(t)$ is a curve for which the speed is 1 at all times and f is a function, then $d/dt f(\vec{r}(t)) = D_{\vec{r}\,'(t)}(f)$.
16)	T	F	Assume f is zero on the boundary of the unit square $R = \{0 \leq x \leq 1, 0 \leq y \leq 1\ \}$, then $\int_0^1 \int_0^1 f_{xy}(x,y)\ dydx = 0$.
17)	T	F	If $f_{yy}(x,y) < 0$ for all x, y, then f can not have any local minimum.
18)	T	F	The double integral $\int_{-1}^1 \int_{-1}^1 x^2 - y^2\ dxdy$ is the volume of the solid below the graph of $f(x,y) = x^2 - y^2$ and above the square $-1 \leq x \leq 1, -1 \leq y \leq 1$ in the xy-plane.
19)	T	F	For any function f and any unit vector $\vec{v}$ one has $D_{\vec{v}}(f) + D_{-\vec{v}}(f) = 0$.

20) [T] [F] The surfaces $x^2 + y^2 + z^2 = 2$ and $x^2 - y^2 + z^2 = 2$ have the same tangent plane at $(1, 0, 1)$.

Problem 2) (10 points) No justifications are needed

a) (6 points) Match the regions with the double integrals. If none applies, put O.



Enter A-F	Integral of Function $f(x,y)$
	$\int_0^{\pi/2} \int_0^{\theta} f(r,\theta) r \, dr d\theta$
	$\int_0^{\pi/2} \int_0^{y} f(x,y) \, dx dy$
	$\int_0^{\pi/2} \int_0^{x} f(x,y) \, dy dx$

Enter A-F	Integral of Function $f(x,y)$
	$\int_0^{\pi/2} \int_{\theta}^{\pi/2} f(r,\theta) r \, dr d\theta$
	$\int_0^{\pi/2} \int_{\pi/2-y}^{\pi/2} f(x,y) \, dx dy$
	$\int_0^{\pi/2} \int_0^{\pi/2-x} f(x,y) \, dy dx$

b) (4 points) Assume $\vec{v} = \nabla f(1,1) = \langle 3/5, 4/5 \rangle$ and $\vec{r}(t) = \langle 1+3t, 1+4t \rangle$ and that $L(x,y)$ is the linearization of f at $(1,1)$ and $f(1,1) = 2$. Finally, denote by $ax + by = d$ the tangent line of f at $(1,1)$.

Here is a choice of 5 answers. 4 of them apply above, fill them in. If none should apply, you would enter O.

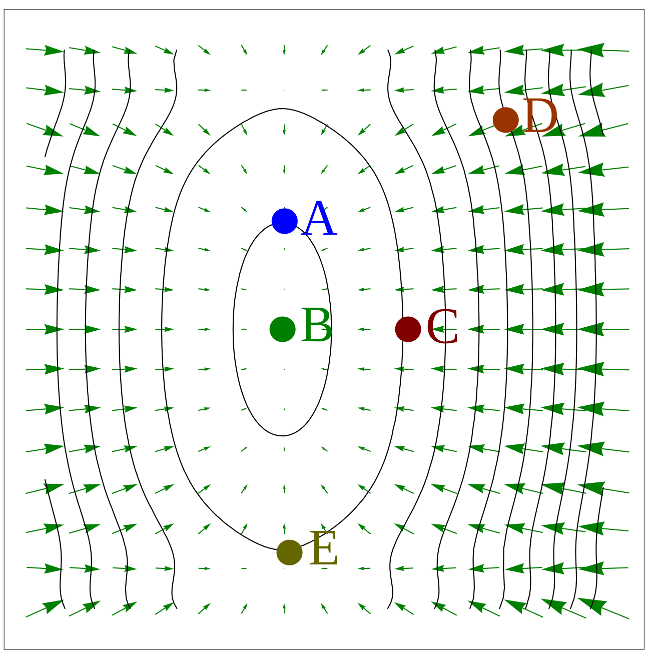
Expression	Fill in A-E
$D_{\vec{v}} f(1,1)$	
$d/dt f(\vec{r}(t))\|_{t=0}$	
$L(2,1)$	
the constant d in the tangent line	

A	5
B	7/5
C	13/5
D	25
E	1

Problem 3) (10 points) (no justifications are needed)

a) (5 points) A function $f(x,y)$ of two variables has level curves as shown in the picture. The arrows are the gradient.

Enter A-E or O if no match	Description
	a local maximum $f(x,y)$.
	a local minimum $f(x,y)$.
	a point, where $f_x = 0$ and $f_y < 0$
	a point, where $f_y = 0$ and $f_x < 0$
	the point among $A - E$, where $\|\|\nabla f\|\|$ is largest



b) (5 points) Assume $f(x,y) = x^2 - y^2 + 1, g(x,y) = x^2 + y^2 + 1$. Which of the following statements are true (T) or false (F):

Enter T/F	Statement
	$(0,0)$ is a critical point for f.
	$(0,0)$ is a critical point for g.
	$(0,0)$ is a critical point for f under the constraint $g = 1$.
	$(0,0)$ is a critical point for g under the constraint $f = 1$
	$(0,0)$ is a global minimum for g.

Problem 4) (10 points)

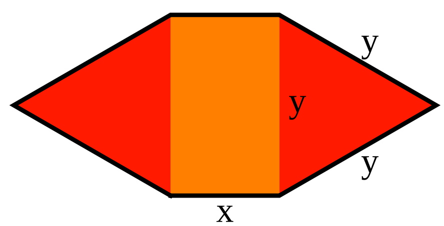
A hexagonal shape is made of two equilateral triangles of side length y and a rectangle of length x and width y. The area is

$$f(x,y) = xy + (\sqrt{3}/2)y^2$$

the circumference is

$$g(x,y) = 2x + 4y \ .$$

Assume the circumference is fixed so that $g(x,y) = 8$. Find the dimensions x, y for which the hexagon has maximal area. Use the method of Lagrange multipliers.



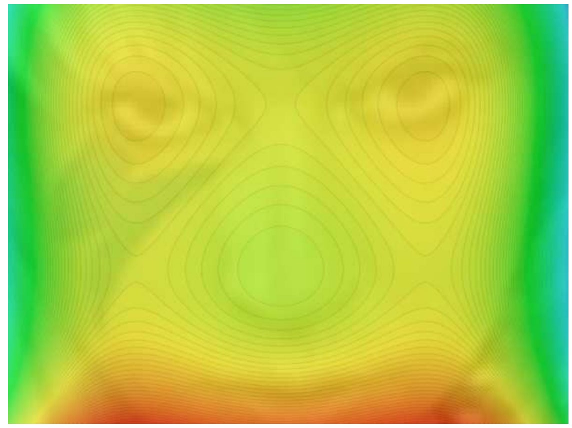
Problem 5) (10 points)

a) (8 points) The face of Lady Gaga - a women of extremes - is modeled by the **Gaga function**

$$f(x,y) = 2y^3 - 3y^2 + x^4 - 2x^2 \ .$$

Classify all critical points of f. Which are maxima, minima or saddle points.

b) (2 points) Is there a global maximum for f or is there no maximal "Gaga"?

Problem 6) (10 points)

Find the surface area of the surface

$$\vec{r}(t,s) = \langle t^2+1, s^2-1, 1+\sqrt{2}ts \rangle \ .$$

for which the parameters satisfy $t^2+s^2 \leq 9$.

Problem 7) (10 points)

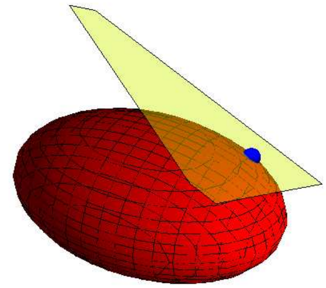
a) (5 points) Find the linearization function $L(x,y)$ of $f(x,y) = \sqrt{x^3 y}$ at $(x,y) = (1,4)$.

b) (5 points) Estimate

$$\sqrt{0.999^3 \cdot 3.9}$$

by evaluating $L(0.999, 3.9)$.

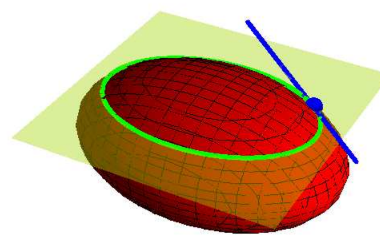
Problem 8) (10 points)



a) (5 points) Find an equation of the form $ax+by+cz = d$ which gives the tangent plane to the ellipsoid

$$4x^2 + 9y^2 + 16z^2 = 41$$

at the point $(2, 1, 1)$.



b) (5 points) If we intersect the ellipsoid with a plane $z = 1$, we obtain an ellipse $g(x, y) = 4x^2 + 9y^2 = 25$. Find an equation of the form $ax + by = d$ for the tangent line to that level curve $g(x, y) = 25$ at the point $(2, 1)$.

Problem 9) (10 points)

Two underwater robots $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ and $\vec{R}(t) = \langle 1 + \sin(t), 1 - \cos(t) \rangle$ circle the just closed BP oil cap in the gulf of Mexico. The oil concentration is an unknown function $f(x, y)$ which is normalized that $f(1, 0) = 0$. The robots measure $D_{\vec{r}\,'(t)} f = 4$ and $D_{\vec{R}\,'(t)} f = 5$ at $t = 0$.

a) (5 points) Find the gradient $\nabla f(1, 0) = \langle a, b \rangle$ of f at $(1, 0)$.

b) (5 points) Estimate the oil concentration $f(1.1, 0.02)$ at the point $(1.1, 0.02)$.