7/25/2013 SECOND HOURLY PRACTICE VIII Maths 21a, O.Knill, Summer 2013

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Provide details to all computations except for problems 1-3.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points)

1) T F It is possible that $(1,1)$ is a local maximum for the function f and $1 = f_{xx} = -f_{yy}$.

2) T F $(0,0)$ is a local maximum of the function $f(x,y) = 5 - x^8 - y^8$.

3) T F If the curvature of a curve is zero everywhere, then it is a line.

4) T F If the Lagrange multiplier λ is negative then the critical point under constraint is a saddle point.

5) T F The arc length of a curve on $[0,1]$ can be obtained by integrating up the curvature of the curve along the interval $[0,1]$.

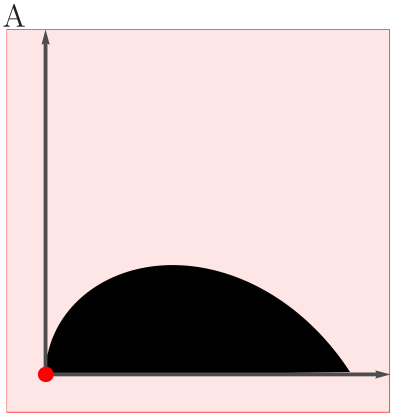
6) T F If D is the discriminant at a critical point and $Df_{xx} > 0$ then we either have a saddle point or a local maximum.

7) T F The function $f(x,y) = \sin(y)x^2 \sin(y^2)$ satisfies the partial differential equation $f_{xyyxyxy} = 0$.

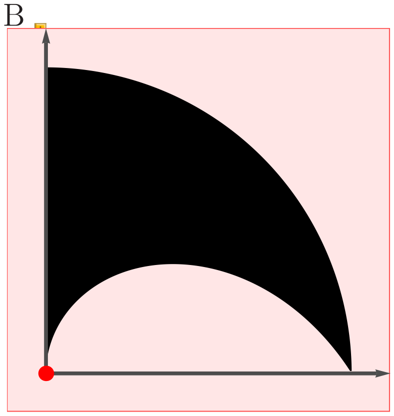
Problem 2) (10 points)

Match the regions with the corresponding polar double integrals

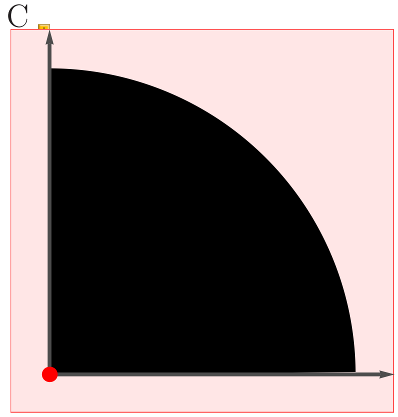
A



B



C



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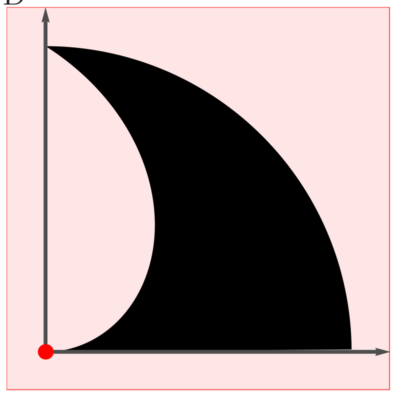

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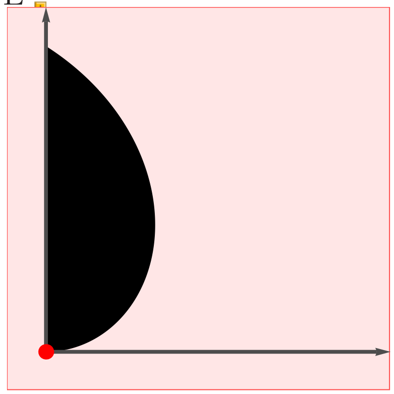
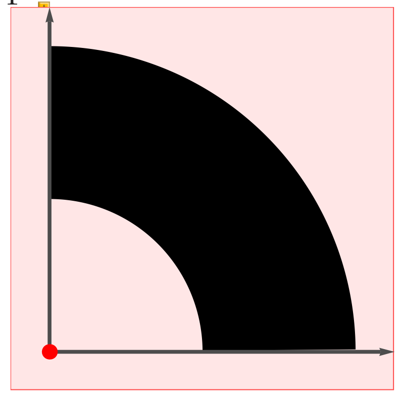


F



Enter A-F	Integral of $f(r,\theta)$
	$\int_0^{\pi/2}\int_0^{\pi/2} f(r,\theta) r\; drd\theta$
	$\int_0^{\pi/2}\int_0^{\theta} f(r,\theta) r\; drd\theta$
	$\int_0^{\pi/2}\int_0^{\pi/2-\theta} f(r,\theta) r\; drd\theta$

Enter A-F	Integral of $f(r,\theta)$
	$\int_0^{\pi/2}\int_\theta^{\pi/2} f(r,\theta) r\; drd\theta$
	$\int_0^{\pi/2}\int_{\pi/2-\theta}^{\pi/2} f(r,\theta) r\; drd\theta$
	$\int_0^{\pi/2}\int_{\pi/4}^{\pi/2} f(r,\theta) r\; drd\theta$

Problem 3) (10 points)

The following statements are not complete. Fill in from the pool of words below.

statement	Fill in the letters	statement
The surface area does		on the parametrization.
$\sqrt{48}$ can be estimated by		at $x = 7$. The result is 7-1/14.
The discriminant D is		if the point is a saddle point.
For a Lagrange minimum, ∇g is		to ∇f.
Arc length is approximated by a		sum if the curve is smooth.
The gradient ∇f is		to the surface $f = c$.

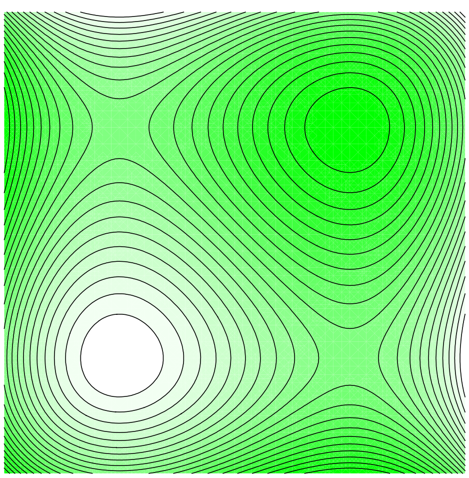
O	not depend
L	linear approximation
I	negative
D	not depend
M	tangent
O	parabola
E	perpendicular
L	parallel
E	orthogonal
R	Rieman

Problem 4) (10 points)

The green near one of the holes in the Cambridge Fresh pond golf course has the height

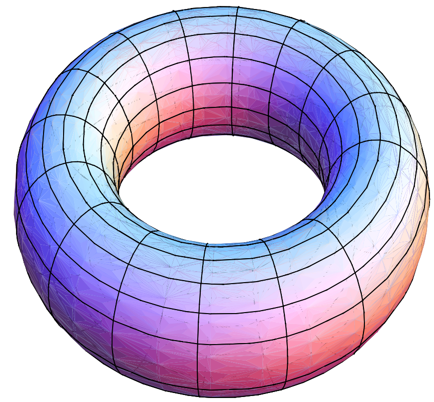
$$f(x, y) = x^3 + y^3 - 3x^2 - 3y^2$$

Find local maxima, local minima or saddle points of this function. Near which point will golf balls most likely end up, if balls like to roll to lower areas.



Problem 5) (10 points)

A torus can be obtained by rotating a circle of radius b around a circle of radius a. The volume of such a torus is $2\pi^2 ab^2$ and the surface area is $4\pi^2 ab$. If we want to find the torus which has minimal surface area while the volume with fixed packing $2\pi^2 a(b^2 + 1)$ is fixed $2\pi^2$, we need to extremize the function $f(a, b) = 4\pi^2 ab$ under the constraint $a + ab^2 = 1$. Find the optimal a, b.



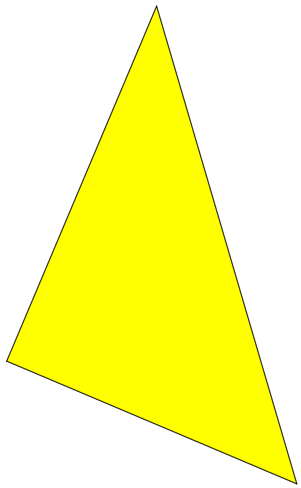
Problem 6) (10 points)

a) Find the arc length of the curve $\vec{r}(t) = \langle t^2, 2t^3/3, 1\rangle$ from $t = -1$ to $t = 1$.
b) What is the curvature of the curve at time $t = 1$? The formula for the curvature is

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

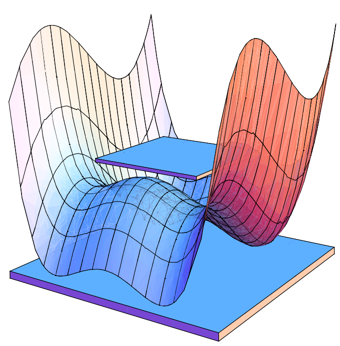
Problem 7) (10 points)

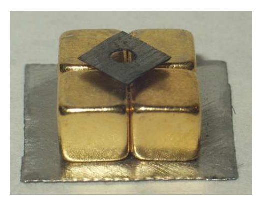
A right angle triangle has the side lengths $x = 0.999$ and $y = 1.00001$. Estimate the value of the hypotenuse $f(x, y) = \sqrt{x^2 + y^2}$ using linear approximation.



Problem 8) (10 points)

Oliver got a diagmagnetic kit, where strong magnets produce a force field in which pyrolytic graphic flots. The gravitational field produces a well of the form $f(x, y) = x^4+y^3-2x^2-3y$. Find all critical points of this function and classify them. Is there a global minimum?





Right picture credit: Wikipedia.