7/20/2017 SECOND HOURLY PRACTICE VIII   Maths 21a, O.Knill, Summer 2017

Name:

- Start by printing your name in the above box.

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or unstaple the packet.

- Provide details to all computations except for problems 1-3.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

|		
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points), no justifications needed

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F — Every critical point of a smooth function $f(x, y)$ of 2 variables is either a maximum or a minimum.

2) T F — If $f(x, y) = 10$ and a region G in the plane is given, then $\iint_G f(x, y)\, dydx$ is 10 times the area of the region.

3) T F — If a function $f(x, y)$ has only one critical point $(0, 0)$ in $G = \{x^2 + y^2 \le 1\}$ which is a local maximum and $f(0, 0) = 1$, then $\iint_G f(x, y)\, dxdy > 0$.

4) T F — If a curve $\vec{r}(t)$ cuts a level curve in a right angle and nonzero velocity at a point which is not critical, then the $d/dt f(r(t)) \ne 0$ at that point.

5) T F — The linearization of a function $f(x, y)$ at $(0, 0)$ has a graph which is a plane $ax + by + cz = d$ tangent to the graph of $f(x, y)$.

6) T F — The surface area $\vec{r}(u, v) = \langle u^2, v^2, u^2 + v^2 \rangle$ with $0 \le u \le 1, 0 \le v \le 1$ is equal to the surface area of $\vec{r}(u, v) = \langle u^3, v^3, u^3 + v^3 \rangle$ with $0 \le u \le 1, 0 \le v \le 1$.

7) T F — Assume $\vec{r}(u, v)$ is a parametrization of a surface $g(x, y, z) = d$ and $\vec{r}(1, 2) = 3$, then $\nabla g(1, 2, 3)$ is parallel to $\vec{r}_u(1, 2) \times \vec{r}_v(1, 2)$.

8) T F — Any region which is both type I and type II must be a rectangle.

9) T F — A given function $f(x, y)$ defines two functions $g(x, y) = f_{xx}(x, y) f_{yy}(x, y) - f_{xy}^2$ and $h(x, y) = f_{xx}(x, y)$. Assume both g and h are positive everywhere, then every critical point of f must be a local minimum.

10) T F — If $f(x, y)$ satisfies the Laplace equation $f_{xx} = -f_{yy}$ then every critical point of f with nonzero discriminant D is a saddle point.

11) T F — The Lagrange multiplier λ at a solution (x, y, λ) of the Lagrange equations $\nabla f(x, y) = \lambda \nabla g(x, y), g(x, y) = 0$ has the property that it is always positive or zero.

12) T F — The partial differential equation $u_t = u_{xx}$ is called the heat equation.

13) T F — The gradient $\nabla f(x, y, z)$ of a function of three variables is a vector tangent to the surface $f(x, y, z) = 0$ if (x, y, z) is on the surface.

14) T F — The value of $\log(2 + x)$ with natural log can be estimated by linear approximation as $\log(2) + x/2$.

15) T F — The tangent line to the curve $f(x, y) = x^3 + y^3 = 9$ at $(2, 1)$ can be parametrized as $\vec{r}(t) = \langle 2, 1 \rangle + t \langle 8, 3 \rangle$ since $\langle 8, 3 \rangle$ is the gradient at $(2, 1)$.

16) T F — Any function $f(x, y)$ which has a local maximum also has a global maximum.

17) T F — The directional derivative of a function $f(x, y)$ in the direction of the tangent vector to the level curve is zero.

18) T F — The directional derivative of a function in the direction $\nabla f / |\nabla f|$ of the gradient is always nonnegative at a point which is not a critical point.

19) T F — The chain rule tells $d/dt f(t^4, t^3)|_{t=1}$ is equal to the dot product of the gradient of f at $(1, 1)$ and the velocity vector $(4, 3)$ at $(1, 1)$.

20) T F — Fubini's theorem assures that $\int_0^1 \int_x^1 f(x, y)\, dxdy = \int_0^1 \int_y^1 f(x, y)\, dydx$.

Problem 2) (10 points) No justifications are needed

Match the regions with the double integrals. Only 5 of the 6 choices match.

a

b

c

d

e

f
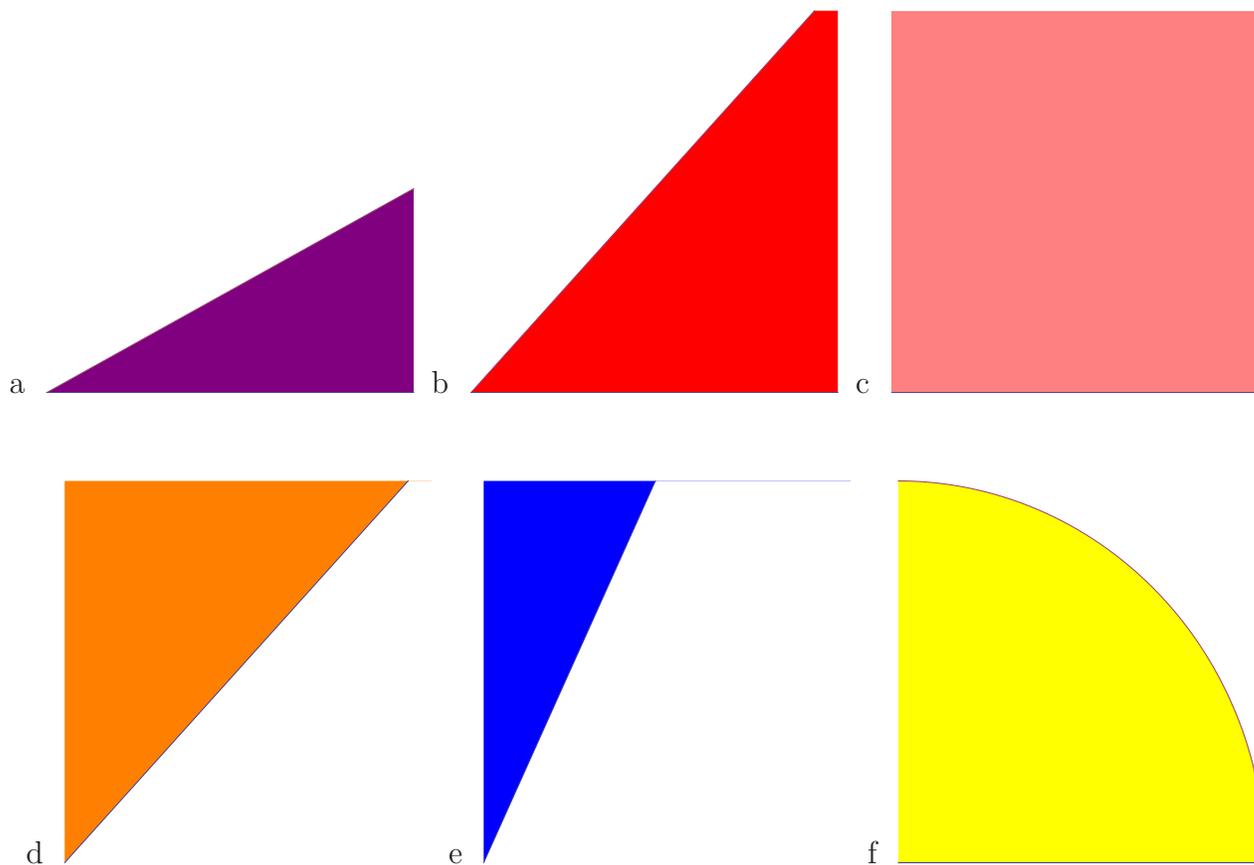
Enter a,b,c,d,e or f	Integral of Function $f(x, y)$
	$\int_0^{\pi/2} \int_0^{\pi/2} f(r, \theta) r \, d\theta dr$
	$\int_0^{\pi/2} \int_0^{\pi/2} f(x, y) \, dxdy$
	$\int_0^{\pi/2} \int_0^{x} f(x, y) \, dydx$
	$\int_0^{\pi/2} \int_0^{y} f(x, y) \, dxdy$
	$\int_0^{\pi/2} \int_0^{x/2} f(x, y) \, dydx$

Problem 3) (10 points) (no justifications are needed)

A function $f(x, y)$ of two variables has level curves as shown in the picture.

| Enter A-E	Description
	a critical point of $f(x, y)$.
	a point, where f is extremal under the constraint $x = 2$.
	a point, where f is extremal under the constraint $y = 0$.
	the point among points A-E, where the length of the gradient vector is largest.
	the point among points A-E, where the length of the gradient vector is smallest.
	a point, where $D_{\langle 1,1\rangle/\sqrt{2}}f = 0$ and $D_{\langle 1,-1\rangle\sqrt{2}}f \neq 0$.
	a point, where $D_{\langle 1,-1\rangle/\sqrt{2}}f = 0$ and $D_{\langle 1,1\rangle/\sqrt{2}}f \neq 0$.
	a point, where $D_{\langle 1,0\rangle}f = 0$ and $D_{\langle 0,1\rangle}f \neq 0$.
	a point, where $f_x = 0$ and $f_y \neq 0$.
	a point, where $f_y = 0$ and $f_x \neq 0$.
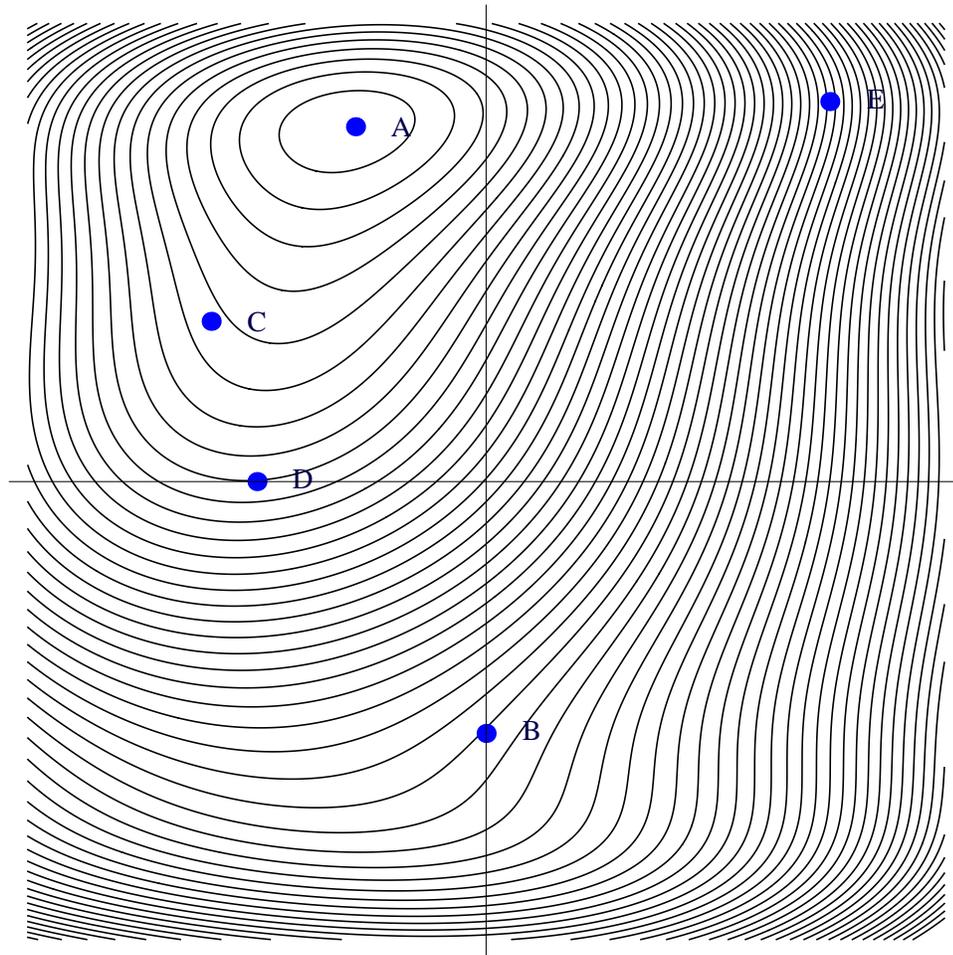
A mass point with position (x, y) is attached by springs to the points $A_1 = (0, 0)$, $A_2 = (2, 0)$, $A_3 = (0, 2)$, $A_4 = (2, 3)$, $A_5 = (3, 1)$. It has the potential energy

$$f(x, y) = 31 - 14x + 5x^2 - 12y + 5y^2$$

which is the sum of the squares of the distances from (x, y) to the 5 points. Find all extrema of f using the second derivative test. The minimum of f is the position, where the mass point has the lowest energy.
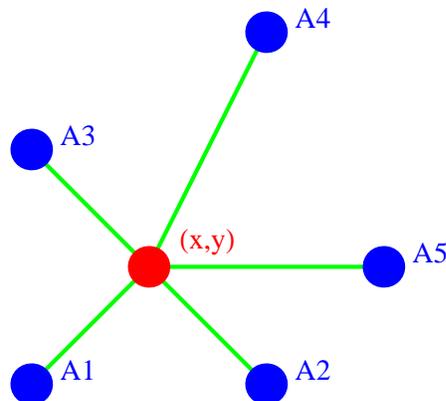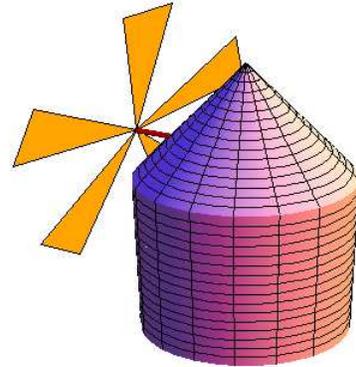
The main building of a mill has a cone shaped roof and cylindrical walls. If the cylinder has radius r, the height of the side wall is h and the height of the roof is $\sqrt{3}r$, then the volume is

$$V(h,r) = \pi r^2 h + h\pi r^2/3 = (4\pi/3)hr^2$$

and area of the building is

$$A(h,r) = \pi r^2 + 2\pi rh + \pi 2r^2 = \pi(3r^2 + 2rh) \ .$$

For fixed volume $V(h,r) = 4\pi/3$, minimize $A(h,r)$ using the Lagrange multiplier method.

a) (5 points) Find the tangent plane to the surface $x^3y + yx^3 + z^2x^2 = 6$ at the point $(1,1,2)$.

b) (5 points) Find the tangent line to the curve $x^4 - y^4 = 15$ at the point $(2,1)$.

a) (4 points) Compute the moment of inertia

$$I = \int \int_G (x^2 + y^2)\,dydx$$

of the half disc $D = \{x^2 + y^2 \leq 1, x \geq 0\ \}$.
b) (6 points) Evaluate the following double integral

$$\int_1^e \int_{\log(x)}^1 \frac{y}{e^y - 1}\,dydx \ ,$$

where log is the natural log as usual.

Problem 8) (10 points)

a) (4 points) Find the linearization $L(x, y)$ of the function $f(x, y) = x^5 \cdot y^3$.

b) (6 points) Estimate $10.01^5 \cdot 4.999^3$ using the linear approximation found in part a).

Problem 9) (5 points)
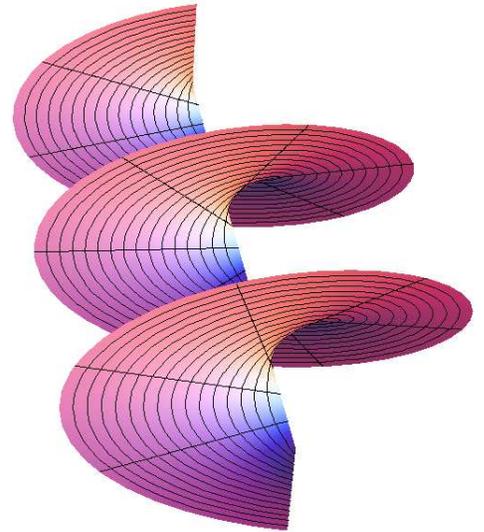
Find the surface area of the surface parametrized by

$$\vec{r}(t, s) = \langle s \cos(t), s \sin(t), t \rangle ,$$

where $0 \leq t \leq 5\pi$ and $0 \leq s \leq 2$.

Hint. You can use the anti derivative formula

$$\int \sqrt{1 + s^2} \, ds = s\sqrt{1 + s^2}/2 + \operatorname{arcsinh}(s)/2$$

computed in class without having to derive it again.

Problem 10) (10 points)

We minimize the surface of a roof of height x and width $2x$ and length $L = \sqrt{2}y$ if the volume $V(x, y) = x^2\sqrt{2}y$ of the roof is fixed and equal to $\sqrt{2}$. In other words, you have to minimize $f(x, y) = 2x^2 + 4xy$ under the constraint $g(x, y) = x^2y = 1$. Solve the problem with the Lagrange method.

L

x

x