On Continuum and Discrete Calculus

Calculus is a theory of **change** and **accumulation** leading to **derivatives** and **integrals**. These ideas work also in the discrete. We illustrate this with a dictionary. 24 topics of our Math S 21a summer school course are compared to notions in **discrete calculus**. The geometric frame-work is **graph theory** or the geometry of **finite abstract simplicial complexes**, a collection of sets. If the sets are the nodes and two sets are connected if one is contained in the other one gets a graph. Think about each set as a **space-time event**.

There are various attempts to combine the quantum world with gravity, none has succeeded yet. Programs **causal sets**, **causal dynamical triangulation** or **loop quantum gravity** illustrate this. Currently there is uncertainty which mathematical structures will matter in a **quantum gravity**, but there is no doubt that any future language describing fundamental processes in nature are **a flavor of calculus**. Whatever this theory will be, multi-variable calculus will remain a prototype structure and remain relevant. Like the Newtonian theory of planetary motion which has been super-seeded fundamentally by **quantum mechanics** or **relativity**, the path of a space craft is still today computed using classical calculus.

Chapter 1. Geometry and Space

space	a three dimensional complex
plane	a two dimensional complex
distance	length of shortest path between two vertices
sphere	all nodes of a fixed distance to a given point

Chapter 2: Curves and Surfaces

scalar function	function on vertices
level surface	the sets where $f - c$ changes sign
arc length	number of edges in a path
surface area	number of triangles in a surface

Chapter 3: Linearization and gradient

gradient	exterior derivative d, a function on edges
curl	exterior derivative d, a function on triangles
div	exterior derivative d, a function on tetrahedra
Laplacian	L=div grad, applies for example to scalar functions

Chapter 4: Extrema and Double integrals

critical point	point a where $\{f(x) < f(a)\}$ is not contractible
discriminant	the index of a critical point
double integral	function over triangles
surface area	number of triangles

Chapter 5: Integrals of fields

vector field	function on oriented edges or triangles
line integral	sum up 1-form F along path
flux integral	sum of 2-form F over surface
triple integral	sum of 3-form F over solid

Chapter 6: Integral theorems

FTLI	defined for a curve a collection of edges
Green	defined on a surface, a collection of triangles
Stokes	surface with boundary in 3-space, defined for collection of triangles
Gauss	region with boundary in 3-space, defined for a collection of tetrahedra

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