7/12/2018 FIRST HOURLY Practice 2 **Maths 21a, O.Knill, Summer 2018**

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Except for problems 1,2 and 8, give details.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) (20 points) No justifications are needed.

1) T F The distance between two points P and Q is equal to length of the vector $\vec{QP}$.

2) T F The symmetric equations of $\vec{r}(t) = [1+t, 2+t, 3+t]^T$ is $x = y = z$.

3) T F The arc length of a circle with constant curvature 6 is $\pi/3$.

4) T F The surface $x^2 + y^2 + 4y = z^2 + 2z$ is a cone.

5) T F A vector is characterized as an entity which has magnitude and direction.

6) T F The Cauchy-Schwartz inequality tells that $|\vec{v} \cdot \vec{w}| \leq |\vec{v} + \vec{w}|$.

7) T F The acceleration of $\vec{r}(t) = [t, t, t]^T$ is zero everywhere.

8) T F If $\vec{v} = \vec{PQ} = [2, 1, 1]^T$ then $|\vec{v}| = d(P, Q)$.

9) T F The surface $x^2 - y^2 + z^2 - 2x = 3$ is a one-sheeted hyperboloid.

10) T F If $\vec{v} \cdot \vec{w}$ is positive, then the angle between $\vec{v}$ and $\vec{w}$ is larger than $90°$.

11) T F There exists a differentiable function for which the level curves $f(x, y) = 1$ and $f(x, y) = 0$ intersect in a point.

12) T F The equation $[1, 0, 1]^T \times \vec{x} = \vec{x}$ has only the zero vector $\vec{x}$ as solution.

13) T F The curvature of $\vec{r}(t) = [t-1, 1-t, t]^T$ is 0 if $t = 1$.

14) T F The line $\vec{r}(t) = t[3, 4, 5]^T$ hits the plane $-4x + 3y = 0$ in a right angle.

15) T F The point given in spherical coordinates as $\rho = 3, \phi = \pi/2, \theta = \pi$ is on the z-axes.

16) T F Given three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, then $|(\vec{u} \cdot \vec{v})\vec{w}| \leq |\vec{u}||\vec{v}||\vec{w}|$.

17) T F The surface given in spherical coordinates as $\rho \cos(\theta) = \rho^2$ is a sphere.

18) T F The arc length of the curve $[\sin(t), 1, \cos(t)]^T$ from $t = 0$ to $t = 2\pi$ is equal to 2π.

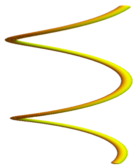
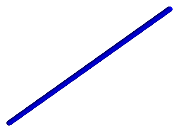
19) T F The surface parametrized by $\vec{r}(u, v) = [u^3 + v^3, u^3 - v^3, u^3]^T$ is a plane.

20) T F If the cross product of a vector $\vec{v}$ with a vector $\vec{w}$ is parallel to $\vec{v}$ then the dot product between $\vec{v}$ and $\vec{w}$ is zero.

Total

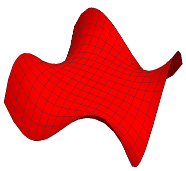
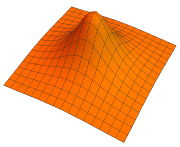
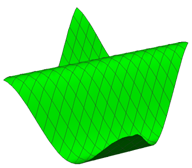
Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the curves. Enter O, if there is no match.

I  II  III 

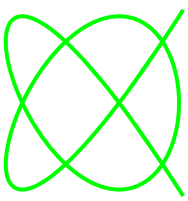
Space curve $\vec{r}(t) =$	Enter O,I,II or III
$[\cos(t), \sin(t), t]^T$	
$[t, t\sin(t), t\cos(t)]^T$	
$[1, t^2\cos(t), t^2\cos(t)]^T$	
$[t, t^2, t^3]^T$	
$[1+t, t, t]^T$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.

I  II  III 

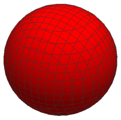
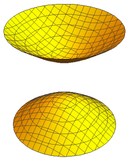
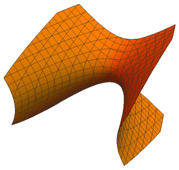
Function $f(x, y) =$	Enter O,I,II or III
$x^3y - y^3x$	
$x^2 + y^2$	
$\cos(x - y)$	
y^3	
$\sin(1/(1 + x^2 + y^2))$	

c) (2 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.

I  II  III 

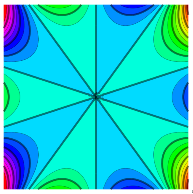
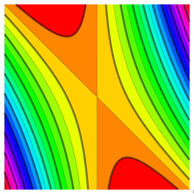
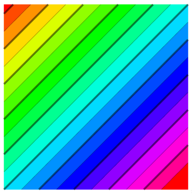
Parametrization $\vec{r}(t) =$	Enter O, I,II or III
$\vec{r}(t) = [t, t^4]^T$	
$\vec{r}(t) = [\cos(4t), \sin(5t)]^T$	
$\vec{r}(t) = [t\cos(5t), t\sin(5t)]^T$	
$\vec{r}(t) = [t^2, t]^T$	
$\vec{r}(t) = [2t, 3t]^T$	

d) (2 points) Match functions g with level surface $g(x, y, z) = 1$. Enter O, if no match.

I  II  III 

Function $g(x, y, z) = 1$	Enter O, I,II or III
$x^2 + y^2 - z^2 = 1$	
$x^2 + y - z^2 = 1$	
$x + y - z^2 = 1$	
$x^2 + y^2 + z^2 = 1$	
$x^2 + y^2 - z^2 = -1$	

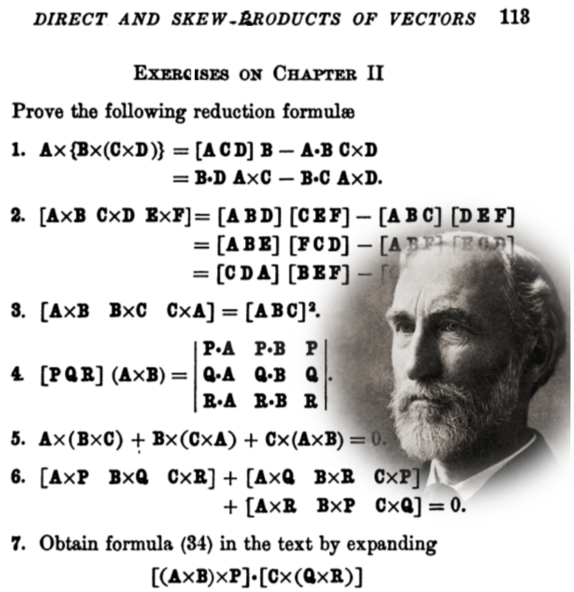
e) (2 points) Match the contour maps to a function $f(x, y)$. Enter O if no match.

I  II  III 

$f(x, y) =$	Enter O,I,II or III
$xy + x^2$	
$x - y$	
x^2	
$x^5 - 10x^3y^2 + 5xy^4$	

Problem 3) (10 points)

Most problems in the multivariable textbook of **Willard Gibbs** and **Edwin Wilson** from 1901 at Yale were proof based. (This book by the way is the prototype of all multi variable textbooks since). Today we like more to compute with concrete vectors. Define $\vec{v} = [4,3,2]^T$ and $\vec{w} = [2,3,4]^T$. Let α be the angle between $\vec{v}$ and $\vec{w}$. No, lets even be more basic and compute some **numbers**:



DIRECT AND SKEW-PRODUCTS OF VECTORS 113

EXERCISES ON CHAPTER II

Prove the following reduction formulæ

1. $\mathbf{A}\times\{\mathbf{B}\times(\mathbf{C}\times\mathbf{D})\} = [\mathbf{A\,C\,D}]\,\mathbf{B} - \mathbf{A}\cdot\mathbf{B}\;\mathbf{C}\times\mathbf{D}$
   $= \mathbf{B}\cdot\mathbf{D}\;\mathbf{A}\times\mathbf{C} - \mathbf{B}\cdot\mathbf{C}\;\mathbf{A}\times\mathbf{D}.$
2. $[\mathbf{A}\times\mathbf{B}\;\;\mathbf{C}\times\mathbf{D}\;\;\mathbf{E}\times\mathbf{F}] = [\mathbf{A\,B\,D}]\,[\mathbf{C\,E\,F}] - [\mathbf{A\,B\,C}]\,[\mathbf{D\,E\,F}]$
   $= [\mathbf{A\,B\,E}]\,[\mathbf{F\,C\,D}] - [\mathbf{A\,B\,F}]\,[\mathbf{E\,C\,D}]$
   $= [\mathbf{C\,D\,A}]\,[\mathbf{B\,E\,F}] - \ldots$
3. $[\mathbf{A}\times\mathbf{B}\;\;\mathbf{B}\times\mathbf{C}\;\;\mathbf{C}\times\mathbf{A}] = [\mathbf{A\,B\,C}]^2.$
4. $[\mathbf{P\,Q\,R}]\,(\mathbf{A}\times\mathbf{B}) = \begin{vmatrix} \mathbf{P}\cdot\mathbf{A} & \mathbf{P}\cdot\mathbf{B} & \mathbf{P} \\ \mathbf{Q}\cdot\mathbf{A} & \mathbf{Q}\cdot\mathbf{B} & \mathbf{Q} \\ \mathbf{R}\cdot\mathbf{A} & \mathbf{R}\cdot\mathbf{B} & \mathbf{R} \end{vmatrix}.$
5. $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0.$
6. $[\mathbf{A}\times\mathbf{P}\;\;\mathbf{B}\times\mathbf{Q}\;\;\mathbf{C}\times\mathbf{R}] + [\mathbf{A}\times\mathbf{Q}\;\;\mathbf{B}\times\mathbf{R}\;\;\mathbf{C}\times\mathbf{P}]$
   $+ [\mathbf{A}\times\mathbf{R}\;\;\mathbf{B}\times\mathbf{P}\;\;\mathbf{C}\times\mathbf{Q}] = 0.$
7. Obtain formula (34) in the text by expanding
   $[(\mathbf{A}\times\mathbf{B})\times\mathbf{P}]\cdot[\mathbf{C}\times(\mathbf{Q}\times\mathbf{R})]$

a) (2 points) Compute $\vec{v}\cdot\vec{w}$.

b) (2 points) Compute $|\vec{v}|\cdot|\vec{w}|$.

c) (2 points) Compute $\cos(\alpha)$.

d) (2 points) Compute $|\vec{v}\times\vec{w}|$.

e) (2 points) Compute $\sin(\alpha)$.

Problem 4) (10 points)

Busy constructions are going on at Harvard this summer. Assume a crane is given by a line

$$L : \vec{r}(t) = [1+t, 2, t]^T \ .$$

John Harvard (rsp. **Sherman Hoar** if you know about the three lies), watches the crane carefully from his location

$$P = (3,4,5) \ .$$

He is very concerned to have his shiny shoe scratched or worse, to have his head knocked off and give us an opportunity to replace the impostor with the real John Harvard. What is the distance between the point P and the line L?

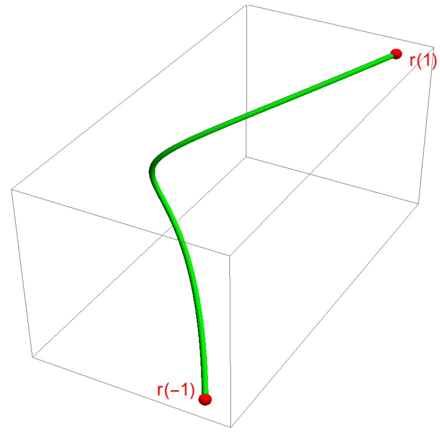
Problem 5) (10 points)

a) (7 points) Find the arc length of the curve

$$\vec{r}(t) = [1 + \sqrt{3}t^2, 1 + 2t^3, 1 + t]^T$$

from $t = -1$ to $t = 1$.

b) (3 points) Evaluate the integral $|\int_{-1}^{1} \vec{r}\,'(t)\;dt|$.

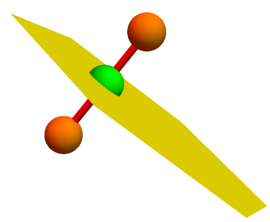


Problem 6) (10 points)

(7 points) Find the equation

$$ax + by + cz = d$$

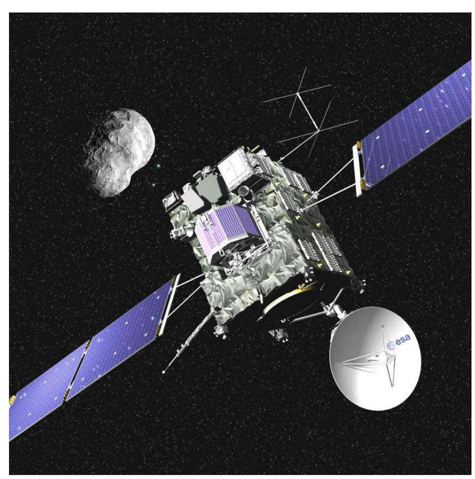
for the plane which passes through the midpoint $M = (P + Q)/2$ of the two points $P = (1, 2, 3)$ and $Q = (3, 4, 5)$. We want the plane to have the property that the line through PQ hits the plane perpendicular at M.



(3 points) What is the distance between P and the plane constructed in a)?

Problem 7) (10 points)

This September 2016, the **Rosetta** spacecraft will make a suicide plunge onto the comet comet Churyumov-Gerasimenk and gather a few last data. Assume its path is $\vec{r}(t) = [t, 2t, 1 - t^2]^T$.



a) (2 points) Find the velocity at $t = 1$.
b) (2 points) Find the acceleration vector at $t = 1$.
c) (2 points) Find the jerk vector $\vec{r}\,'''(t)$ at $t = 1$.
d) (2 points) Give the unit tangent vector at $t = 1$.
e) (2 points) Compute the curvature

$$\frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

at $t = 1$.

Problem 8) (10 points)

We enjoy the summer and ask in a local restaurant for a refreshment. In each of the ordered items, give a **surface parametrization** of the form

$$\vec{r}(u,v) = [x(u,v), y(u,v), z(u,v)]^T .$$

As indicated, we can use also other variables. Your task is to fill in the three parametrization functions in each case, using the variables provided.

a) (2 points) Parametrize the **lemonade glass** $x^2 + y^2 = 1$.

$\vec{r}(\theta, z) = [\boxed{}, \boxed{}, \boxed{}]^T$.

b) (2 points) Parametrize the **sorbet glass** $x^2+y^2 = z^2$.

$\vec{r}(\theta, z) = [\boxed{}, \boxed{}, \boxed{}]^T$.

c) (2 points) Parametrize the **lemon** surface $x^2 + y^2 = \sin(z)$.

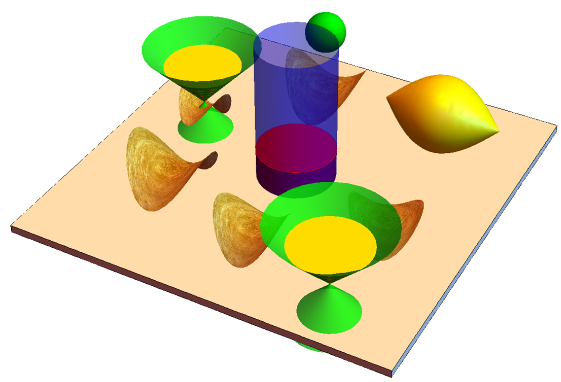
$\vec{r}(\theta, z) = [\boxed{}, \boxed{}, \boxed{}]^T$.

d) (2 points) Parametrize one of the **chips** $z = x^2 - y^2$.

$\vec{r}(x, y) = [\boxed{}, \boxed{}, \boxed{}]^T$.

e) (2 points) Parametrize the **lime** $x^2+y^2+(z-3)^2 = 1$.

$\vec{r}(\theta, \phi) = [\boxed{}, \boxed{}, \boxed{}]^T$.



Problem 9) (10 points)

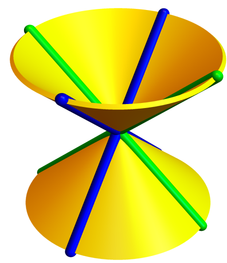
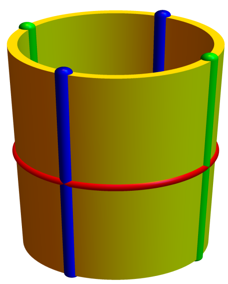
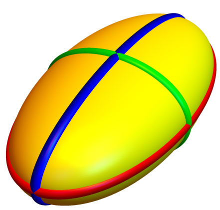
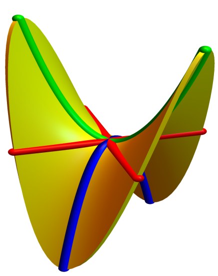
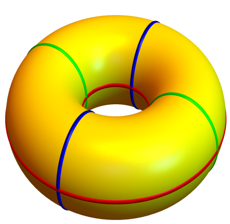
The **Juno space craft** has arrived at Jupiter on July 4. It is an exciting and dangerous journey. For example: since the magnetic field of Jupiter is 20'000 times stronger than the earth's magnetic field, it required the electronics to be shielded from radiation. The acceleration on the space craft is given by

$$\vec{r}\,''(t) = [t, t^2, 1-t]^T .$$

We know that the position at $t = 0$ is given by $\vec{r}(0) = (3, 4, 5)$ when $t = 0$. Additionally, we know that its velocity at time $t = 1$ is $\vec{r}\,'(1) = [1, 0, 0]^T$. (Note $t = 1$ for velocity measurement and $t = 0$ for initial position are different!)
Find the position of the space craft at time $t = 3$.

Problem 10) (10 points)

The 3D printing venture "Math-Candy" (math-candy.com) asks you to do some product development. In each of the 5 following parametrizations, two entries are still missing, each entry being worth one candy (1 point).

a)		The surface $x^2 + y^2 = z^2$ is parametrized by $\vec{r}(\theta, z) = [\ldots\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots\ldots, z]^T$
b)		The surface $x^2 + y^2 = 1$ is parametrized by $\vec{r}(\theta, z) = [\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots, z]^T$
c)		The surface $2(x-1)^2 + (y-5)^2 + 4z^2 = 1$ is parametrized by $\vec{r}(\theta, \phi) = [\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots, \cos(\phi)/2]^T$
d)		The surface $x^2 - y^2 = z$ is parametrized by $\vec{r}(x, y) = [\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots, x^2 - y^2]^T$
e)		The surface $(\sqrt{x^2 + y^2} - 2)^2 + z^2 = 1$ is parametrized by $\vec{r}(\theta, \phi) =$ $[(2 + \cos(\phi))\cos(\theta), \ldots\ldots\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots]^T$