

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Except for problems 1-3 give details.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

- 1)

T	F
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 The length of the vector $\vec{v} \times \vec{v}$ is 3, if $\vec{v} = [1, 1, 1]^T$.
- 2)

T	F
---	---

 For any 4 points A, B, C, D in space, there is a plane which goes through these points.
- 3)

T	F
---	---

 For any two points P, Q in space which have positive distance there is exactly one plane which has equal distance to both points.
- 4)

T	F
---	---

 The graph of a function $f(x, y)$ can be parametrized as $\vec{r}(x, y, z) = z - f(x, y)$.
- 5)

T	F
---	---

 The graph of the function $f(x, y) = x^2 - y^2$ is called a hyperboloid.
- 6)

T	F
---	---

 The equation $\rho \cos(\theta) \sin(\phi) = 1$ in spherical coordinates defines a plane.
- 7)

T	F
---	---

 The vector $[1, 2]^T$ is perpendicular to the line $x + 2y = 2$.
- 8)

T	F
---	---

 The triple scalar product between the vectors $\vec{i}, [0, 2, 0]^T$ and $[1, 1, 1]^T$ is 2.
- 9)

T	F
---	---

 The two parametrized curves $\vec{r}(t) = [t, t^2, t^6]^T, 0 \leq t \leq 2$ and $\vec{R}(t) = [t^2, t^4, t^{12}]^T, 0 \leq t \leq 2$ have the same curvature at $t = 1$.
- 10)

T	F
---	---

 The point $(x, y, z) = (1, 1, 1)$ has the spherical coordinates $(\rho, \theta, \phi) = (\sqrt{3}, \pi/2, \pi/4)$.
- 11)

T	F
---	---

 The distance between the cylinders $x^2 + z^2 = 1$ and $x^2 + (y - 3)^2 = 1$ is 1.
- 12)

T	F
---	---

 If P is the projection onto the line spanned by a vector $\vec{w}$ then $P(\vec{w}) = \vec{w}$.
- 13)

T	F
---	---

 The vector projection of $[2, 3, 1]^T$ onto $[1, 1, 1]^T$ is parallel to $[1, 1, 1]^T$.
- 14)

T	F
---	---

 The triple scalar product has the property $\vec{u} \cdot (\vec{v} \times \vec{u}) = 0$ for any two vectors $\vec{u}, \vec{v}$.
- 15)

T	F
---	---

 For any two vectors $\vec{v}$ and $\vec{w}$, the length of the cross product $\vec{v} \times \vec{w}$ is equal to the length of $\vec{v} + \vec{w}$.
- 16)

T	F
---	---

 If the domain of a function is the entire plane, then the range of the function is the entire line or a half line.
- 17)

T	F
---	---

 There are two unit vectors $\vec{v}$ and $\vec{w}$ which have the property that $\vec{v} \cdot \vec{w} = \pi$.
- 18)

T	F
---	---

 If we travel with unit speed on a street, then the curvature $\kappa(\vec{r}(t))$ of the street is bounded by the magnitude of the acceleration $|\vec{r}''(t)|$.
- 19)

T	F
---	---

 The curve $\vec{r}(t) = [\cos(2t) \sin(3t), \sin(2t) \sin(3t), \cos(3t)]^T$ is located on a sphere.
- 20)

T	F
---	---

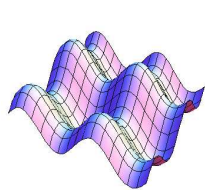
 The surface $x^2 - y^2 + z^2 = 4z - 3$ is a one sheeted hyperboloid.

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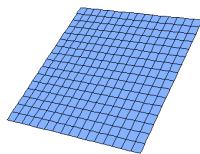
 Total

Problem 2) (10 points) No justifications are needed in this problem.

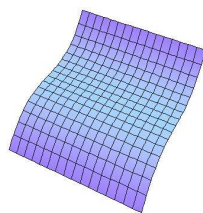
a) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.



I



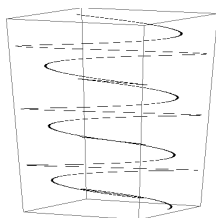
II



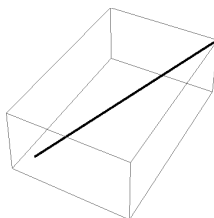
III

Function $f(x, y) =$	Enter O,I,II or III
y^3	
$\sin(x) + \sin(y)$	
$x^2 - y^2$	
$x^4 - y^4$	
$x + y$	

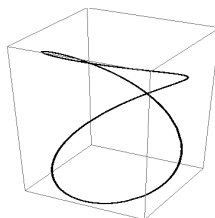
b) (3 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.



I



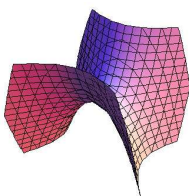
II



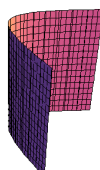
III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\vec{r}(t) = [\cos(2t), \sin(3t), t]^T$	
$\vec{r}(t) = [t , t , t^2]^T$	
$\vec{r}(t) = [\sin(2t)/2, \sin^2(t), \cos(t)]^T$	
$\vec{r}(t) = [1 + 2t, 2 + 3t, t]^T$	

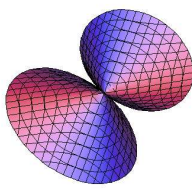
c) (2 points) Match the functions g with the level surface $g(x, y, z) = 1$. Enter O, where no match.



I



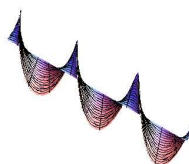
II



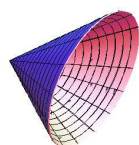
III

Function $g(x, y, z) =$	O, I,II,III
$g(x, y, z) = x^2 + z = y^2$	
$g(x, y, z) = x^2 - y^2 = 1$	
$g(x, y, z) = x^2 - y^2 + z^2 = 0$	
$g(x, y, z) = \frac{x^2}{2} + 2y^2 + z^2 = 1$	

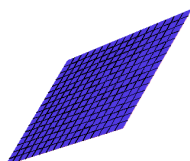
d) (3 points) Match the surface with the parametrization. Enter O, where no match.



I



II



III

Function $g(x, y, z) =$	O,I,II,III
$\vec{r}(s, t) = [t, s, ts]^T$	
$\vec{r}(s, t) = [t - 1, s, t + s]^T$	
$\vec{r}(s, t) = [t, s \cos(t), s \sin(t)]^T$	
$\vec{r}(s, t) = [s, s \cos(t), s \sin(t)]^T$	

Problem 3) (10 points) No detailed justifications are needed.

a) (6 points) We are given a point P , a line L determined by A and $\vec{v}$ a line M determined by B and $\vec{w}$ and a plane Σ . Match the distances with distance formulas and possible explanations:

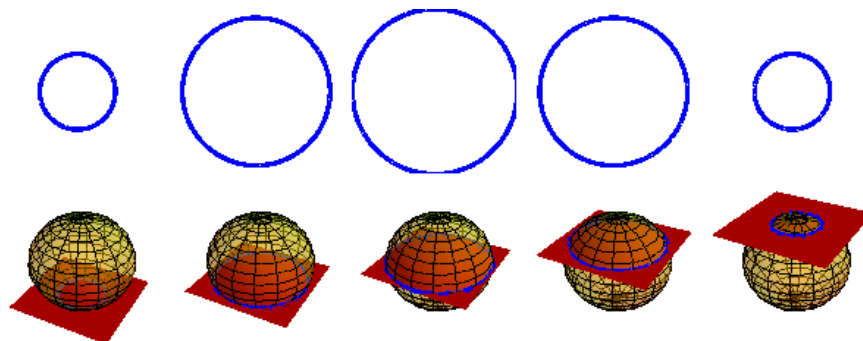
distance	the formula involves (A-C)	and can be explained by (U-W)
$d(P, \Sigma)$		
$d(L, M)$		
$d(P, L)$		

- | | |
|-------------------------------------|---|
| A) triple scalar product | U projection of a connection vector |
| B) dot product and no cross product | V volume of parallelepiped divided by base area |
| C) cross product only | W area of parallelogram divided by base length |

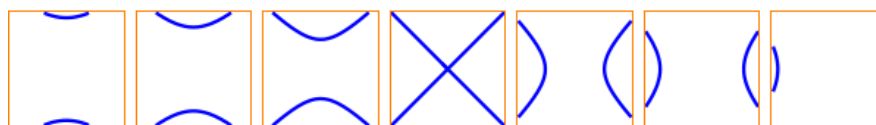
b) (4 points total) Edwin Abbott's novella "Flatland, A Romance of Many dimensions" deals with "flatlanders" who can not perceive three dimensional objects. By looking at various traces $f = c$ a flatlander still can see a three dimensional object. For example, a three dimensional sphere is visualized as follows.



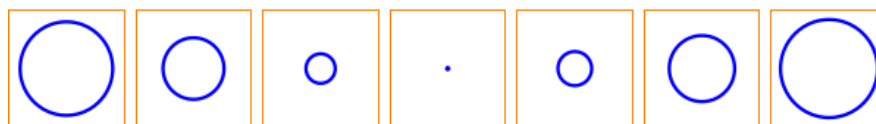
Edwin Abbott: 1838-1926



b1) (2 points) We are in flatland, where multivariable calculus students only work with 2 dimensions. To get a feel for the third dimension the following sequence of generalized traces are shown with a frame around each picture. They illustrate the graph of a function of two variables. Which surface is under consideration?

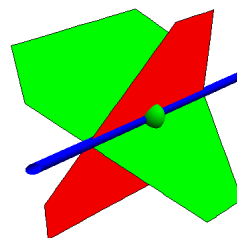


b2) (2 points) In the first midterm exam of that multivariable calculus course, the class has to find the name of the surface of the following sequence of traces. Can you do it?



Problem 4) (10 points)

Find the equation $ax + by + cz = d$ of a plane Σ perpendicular to the planes $x + y - z = 1$ and $x + 2y - 3z = 0$ so that the surface Σ contains the point $P = (2, 1, 2)$.

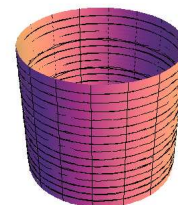
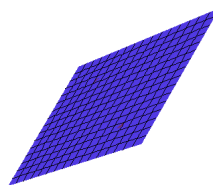
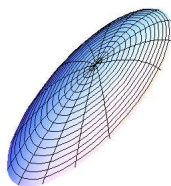
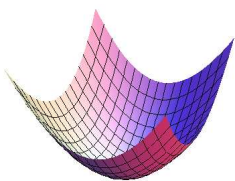


Problem 5) (10 points)

In this problem we find some parametrizations of surfaces which is of the form

$$\vec{r}(u, v) = [x(u, v), y(u, v), z(u, v)]^T.$$

- a) (2 points) Parametrize the **paraboloid** $z = x^2 - y^2$.
- b) (3 points) Parametrize (the entire!) **ellipsoid** $(x - 1)^2 + \frac{(y-2)^2}{4} + z^2 = 1$.
- c) (2 points) Parametrize the **plane** $x + y + z = 3$.
- d) (3 points) Parametrize the **cylinder** $x^2 + z^2 = 1$.



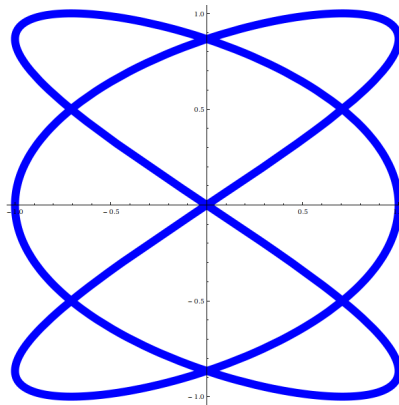
Problem 6) (10 points)

- a) (6 points) Find the velocity vectors $\vec{r}'(\pi/2)$ and $\vec{r}'(3\pi/2)$. Now find the angle between these vectors at the point, where the curve

$$\vec{r}(t) = [\cos(3t), \sin(2t)]^T$$

self-intersects at the origin.

- b) (4 points) Find $\vec{r}'(\pi/2)$, $\vec{r}''(\pi/2)$ and $\vec{r}'(\pi/2) \times \vec{r}''(\pi/2)$. What is the curvature $|\vec{r}'(\pi/2) \times \vec{r}''(\pi/2)|/|\vec{r}'(\pi/2)|^3$ at $(0,0)$?



Problem 7) (10 points)

Compute the following expressions:

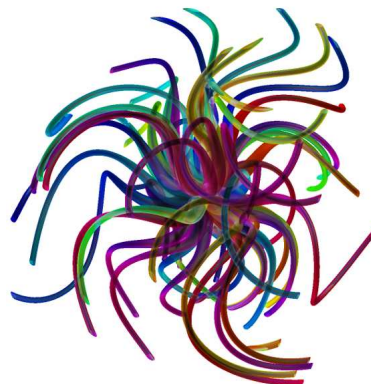
- a) (2 points) the length of the vector $[12, 5]^T$,
b) (2 points) the cross product $[1, 2]^T \times [5, 1]^T$,
c) (2 points) the dot product $[1, 4]^T \cdot [2, 2]^T$,
d) (2 points) the projection $P_{\vec{w}}([1, 2]^T)$ onto the x axes spanned by $\vec{w} = [1, 0]^T$.
e) (2 points) the angle between $[1, 1]^T$ and $[0, 1]^T$.

Problem 8) (10 points)

Oliver attempted to build some of Chihuly's sculptures in Mathematica. To get the organic forms, spiral curves of the form

$$\vec{r}(t) = [\sin(t) \cos(t), \sin(t) \sin(t), t]^T$$

were used. Find the arc length of this curve from $0 \leq t \leq \pi$.



Problem 9) (10 points)

To celebrate **Bastille day** today on the 14th of July, we fire some fireworks. The rocket experiences during the firing phase a time dependent acceleration

$$\vec{r}''(t) = [0, 0, 100 \cos(t) - 10]^T$$

during the rocket firing time $[0, \pi/2]$. It lifts off at $\vec{r}(0) = [1, 0, 0]^T$ with zero velocity $\vec{r}'(0) = [0, 0, 0]^T$. Where will the rocket be at time $t = \pi/2$?

