Name:

- Start by writing your name in the above box.

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have exactly 90 minutes to complete your work.

|		
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

1) T F The length of the vector $[1, 1, 1]^T$ is equal to 3.

2) T F Any two distinct points A, B in space determine a unique line which contains these two points.

3) T F For any two non-intersecting lines L, K, there is exactly one point P which has equal distance to both lines.

4) T F The graph of $f(x, y)$ is a surface in space which is equal to the level surface $g(x, y, z) = 0$ of $g(x, y, z) = f(x, y) - z$.

5) T F The graph of the function $f(x, y) = x^2 - y^2$ is called an elliptic paraboloid.

6) T F The equation $\rho \cos(\theta) = 1$ in spherical coordinates defines a plane.

7) T F The vector $[1, 2, 3]^T$ is perpendicular to the plane $x + 2y + 3z = 4$.

8) T F The cross product between the vectors $[1, 2, 3]^T$ and $[1, 1, 1]^T$ is 6.

9) T F The two parametrized curves $\vec{r}(t) = [t, t^2, t^6]^T, 0 \le t \le 1$ and $\vec{R}(t) = [t^2, t^4, t^6]^T, 0 \le t \le 1$ have the same arc length.

10) T F The point $(1, 0, 1)$ has the spherical coordinates $(\rho, \theta, \phi) = (\sqrt{2}, 0, \pi/4)$.

11) T F The distance between two parallel planes is the distance of any point on one plane to the other plane.

12) T F $\text{Proj}_{\vec{w}}(\vec{v} \times \vec{w}) = \vec{0}$ holds for all nonzero vectors $\vec{v}, \vec{w}$.

13) T F The vector projection of $[2, 3, 4]^T$ onto $[1, 0, 0]^T$ is $[2, 0, 0]^T$.

14) T F The triple scalar product $\vec{u} \cdot (\vec{v} \times \vec{w})$ between three vectors $\vec{u}, \vec{v}, \vec{w}$ is zero if and only if two or more of the 3 vectors are parallel.

15) T F There are two vectors $\vec{v}$ and $\vec{w}$ so that the dot product $\vec{v} \cdot \vec{w}$ is equal to the length of the cross product $|\vec{v} \times \vec{w}|$.

16) T F Two cylinders of radius 1 whose axes are lines L, K have distance $d(L, K) = 3$ have distance 2.

17) T F There are two unit vectors $\vec{v}$ and $\vec{w}$ that are both parallel and perpendicular.

18) T F Assuming the curvature to exist for all time, the curvature $\kappa(\vec{r}(t))$ is always smaller than or equal to $|\vec{r}''(t)|/|\vec{r}'|^2$.

19) T F The curve $\vec{r}(t) = [\cos(t) \sin(t), \sin(t) \sin(t), \sin(t)]^T$ is located on a sphere.

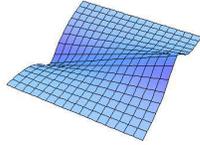20) T F The surface $x^2 + y^2 + z^2 = 4z - 3$ is a sphere of radius 1.

Total

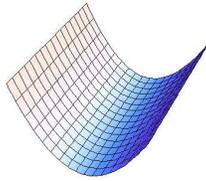Problem 2) (10 points) No justifications are needed in this problem.

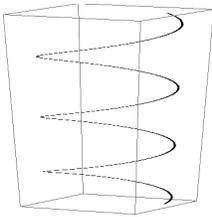a) (2 points) Match the graphs of the functions $f(x,y)$. Enter O, if there is no match.
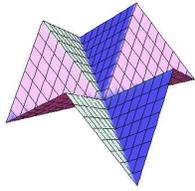
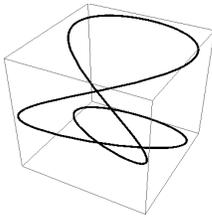| Function $f(x,y) =$	Enter O,I,II or III
x^2	
$||x| - |y||$	
$x^2 + y^2$	
$x/(1 + y^2)$	

I   II   III
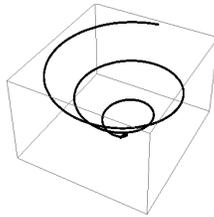
b) (3 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.
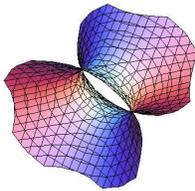
Parametrization $\vec{r}(t) =$	O, I,II,III
$\vec{r}(t) = [t\cos(t), t\sin(t), t]^T$	
$\vec{r}(t) = [\cos(t), \sin(t), t]^T$	
$\vec{r}(t) = [\sin(t), \cos(t), 0]^T$	
$\vec{r}(t) = [\sin(3t), \cos(2t), \cos(t)]^T$	

I   II   III
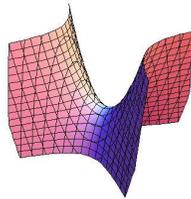
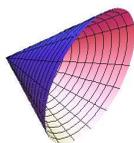c) (2 points) Match the functions g with the level surface $g(x,y,z) = 1$. Enter O, where no match.

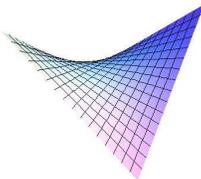| Function $g(x,y,z) = 1$	O, I,II,III
$g(x,y,z) = x^2 - y^2 - z = 1$	
$g(x,y,z) = x - y^2 = 1$	
$g(x,y,z) = x^2 - y^2 + z^2 = 1$	
$g(x,y,z) = x^2 + y^2 + z^2 = 1$	

I   II   III

d) (3 points) Match the surface with the parametrization. Enter O, where no match.

Function $g(x,y,z) =$	O,I,II,III
$\vec{r}(s,t) = [t, s, ts]^T$	
$\vec{r}(s,t) = [t^2 + s^2, s, t]^T$	
$\vec{r}(s,t) = [t, t\cos(s), t\sin(s)]^T$	
$\vec{r}(s,t) = [t, s, t]^T$	

I   II   III

Problem 3) (10 points) No justifications are needed in this problem.

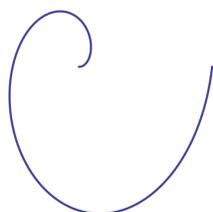a) (2 points) Translate from polar to Cartesian coordinates or back:

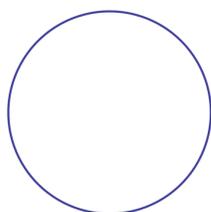| Polar coordinates $(\theta, r) =$	Cartesian coordinates $(x, y) =$
$(\pi/2, 1)$	
	$(1, 1)$

b) (2 points) Translate from spherical to Cartesian coordinates or back:

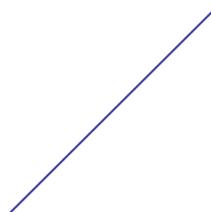| Spherical coordinates $(\theta, \phi, \rho) =$	Cartesian coordinates $(x, y, z) =$
$(\pi/2, \pi/2, 1)$	
	$(1, 1, 1)$

c) (3 points) Match the curves given in polar coordinates. Enter O, if there is no match.
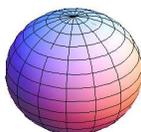

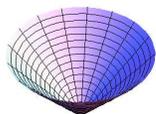I   II   III

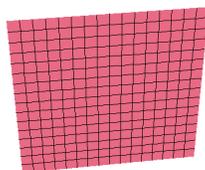| Surface	Enter I,II,II, O
$r = \pi/4$	
$r = -\theta$	
$\theta = \pi/4$	
$r = \theta$	

d) (3 points) Match the surfaces given in spherical coordinates. Enter O, if there is no match.

I   II   III

Surface	Enter I,II,III,O
$\rho = \pi/4$	
$\phi = \pi/4$	
$\theta = \phi$	
$\theta = \pi/4$	

Problem 4) (10 points)

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a) (2 points) Given two points $A = (1, 2, 3)$ and $B = (4, 5, 6)$. Find the midpoint M between these two points.

b) (5 points) Find the equation $ax + by + cz = d$ of the plane for which every point has equal distance to both A and B.

c) (3 points) Write down a parametrization $\vec{r}(t) = [x(t), y(t), z(t)]^T$ of the line containing both A and B.

Problem 5) (10 points)

In this problem you have to find parametrizations of surfaces. The parametrization should have the form

$$\vec{r}(u, v) = [x(u, v), y(u, v), z(u, v)]^T .$$

a) (2 points) Parametrize the paraboloid $z = -x^2 - y^2$.

b) (2 points) Parametrize the ellipsoid $x^2 + y^2/4 + z^2/9 = 1$.

c) (2 points) Parametrize the plane $x = y$.

d) (2 points) Parametrize the cylinder $x^2 + z^2 = 9$.

e) (2 points) Parametrize the cone $x^2 + y^2 = z^2$.

Problem 6) (10 points)

In the soccer world championship of 2010, the English team scored a goal against the German team which the referee did not see. Assume the ball followed the line $x = y = (z - 1)/3$ and that the referee was at position $R = (3, 2, 1)$.

Remark: The incidence was a Wimbleton revanche and confirmed a word of wisdom of former player Gary Lineker who said a couple of years ago: "Soccer is a game for 22 people that run around, play the ball, and one referee who makes a slew of mistakes, and in the end, Germany always wins."

a) (4 points) Find a parametrization of the line L which the ball followed.

b) (6 points) Find the minimal distance of L from the point R.

Problem 7) (10 points)

Compute the following expressions:

a) (2 points) the length of the vector $[1, 1, 2]^T$,

b) (2 points) the cross product $[1, 1, 2]^T \times [2, 2, 1]^T$,

c) (2 points) the dot product $[1, 1, 4]^T \cdot [2, 4, 2]^T$,

d) (2 points) the projection $\text{Proj}_{[1,1,2]^T}([1, 2, 3]^T)$,

e) (2 points) the angle between $[1, 1, 0]^T$ and $[0, 1, 1]^T$.

Problem 8) (10 points)

a) (3 points) Find the unit tangent vector $\vec{T}(t)$ of the curve

$$\vec{r}(t) = [t^2, 3t^3, 12t^{5/2}/5]^T$$

at time $t = 1$.

b) (7 points) Find the arc length of the same curve from $0 \le t \le 2$.

Problem 9) (10 points)

In "Avatar", the "floating mountains of Pandora" contain material which is attracted by the neighboring planet. These stones would therefore float away if not tied back by tree branches or weighted with usual rocks. Pandora stones feel a force (= acceleration) $\vec{r}\,''(t) = [0, 0, 2]^T$. Neytiri throws such a stone from the initial position $\vec{r}(0) = [1, 0, 3]^T$ with initial velocity $\vec{r}\,'(0) = [3, 4, 0]^T$ towards the bottom S of a floating rock $z = 100$.

a) (5 points) Find the path $\vec{r}(t) = [x(t), y(t), z(t)]^T$ of the stone.

b) (2 points) At which time t does the stone hit the rock wall S given by the equation $z = 100$?

c) (3 points) What is the angle of impact between the stone path and the normal vector $[0, 0, 1]^T$ of S?