7/26/2018 2. HOURLY PRACTICE XII **Maths 21a, O.Knill, Summer 2018**

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F $(0,0)$ is a local minimum of the function $f(x,y) = x^6 + y^6$.

2) T F If $f(x,y)$ has a local max at the point $(0,0)$ with discriminant $D > 0$, then $g(x,y) = f(x,y) - x^9 + y^9$ has a local max at $(0,0)$ too.

3) T F The value of the function $f(x,y) = \sqrt{1+3x+5y}$ at $(-0.002, 0.01)$ can by linear approximation be estimated as $1 - (3/2) \cdot 0.002 + (5/2) \cdot 0.01$.

4) T F The value of the double integral $\int_0^{\pi/4} \int_0^2 x^3 \cos(y) \; dxdy$ is the same as $(\int_0^2 x^3 \; dx)(\int_0^{\pi/4} \cos(y) \; dy)$.

5) T F The chain rule can be written in the form $\frac{d}{dt} f(\vec{r}(t)) = D_{\vec{r}\,'(t)} f(\vec{r}(t))$

6) T F The gradient of f at a point (x_0, y_0, z_0) is tangent to the level surface of f which contains (x_0, y_0, z_0).

7) T F If the Lagrange multiplier λ is positive, then the critical point under constraint is a local minimum.

8) T F If the directional derivative $D_{\vec{v}} f(1,1) = 0$ for all vectors $\vec{v}$, then $(1,1)$ is a critical point of $f(x,y)$.

9) T F For any curve $\vec{r}(t)$, the vectors $\vec{r}\,''(t)$ and $\vec{r}\,'(t)$ are always perpendicular to each other.

10) T F Every critical point (x,y) of a function $f(x,y)$ for which the discriminant D is not zero is either a local maximum or a a local minimum.

11) T F The function $f(x,y) = e^y x^2 \sin(y^2)$ satisfies the partial differential equation $f_{xxyyyxyy} = 0$.

12) T F If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) < 0$ then $(0,0)$ can not be a local minimum.

13) T F In the second derivative test, one can replace the condition $D > 0, f_{xx} > 0$ with $D > 0, f_{yy} > 0$ to check whether a point is a local minimum.

14) T F If $\vec{r}(t)$ is a curve in space and f is a function of three variables, then $\frac{d}{dt} f(\vec{r}(t)) = 0$ for $t = 0$ implies that $\vec{r}(0)$ is a critical point of $f(x,y)$.

15) T F The function $f(x,y) = (x^4 - y^4)$ has neither a local maximum nor a local minimum at $(0,0)$.

16) T F If every point of the plane is a critical point for a function f then f is a constant function.

17) T F If $f(x,y)$ has a local max at the point $(0,0)$ with discriminant $D > 0$, then $g(x,y) = f(x,y) - x^9 + y^9$ has a local max at $(0,0)$ too.

18) T F The value of the function $f(x,y) = \log(e + 3x + 5y)$ at $(-0.002, 0.01)$ can by linear approximation be estimated as $1 - 0.006 + 0.05$.

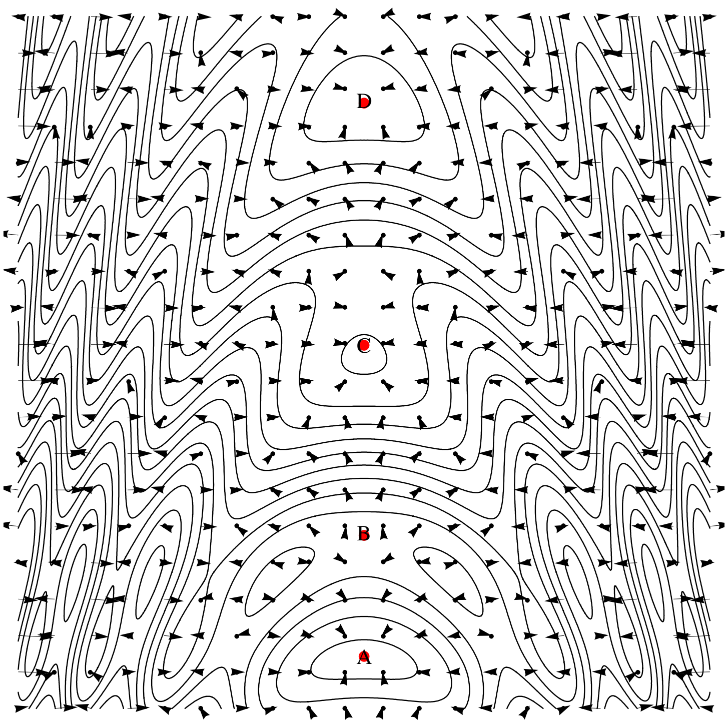
19) T F The chain rule tells that $\frac{d}{dt} f(\vec{r}(t)) = f(\vec{r}(t)) r'(t)$

20) T F If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.

Problem 2) (10 points)

a) (5 points) The picture below shows the contour map of a function $f(x, y)$ which has many critical points. Four of them are outlined for you on the y axes and are labeled A, B, C, D and ordered in increasing y value. The picture shows also the gradient vectors. Determine from each of the 4 points whether it is a local maximum, a local minimum or a saddle point. No justification is necessary in this problem.

Point	Max	Min	Saddle
D			
C			
B			
A			



b) (5 points)

Match the integrals with those obtained by changing the order of integration. No justifications are needed. Note that one of the Roman letters I)-V) will not be used, you have to chose four out of five.

Enter I,II,III,IV or V here.	Integral
	$\int_0^1 \int_{1-y}^1 f(x,y)\, dxdy$
	$\int_0^1 \int_y^1 f(x,y)\, dxdy$
	$\int_0^1 \int_0^{1-y} f(x,y)\, dxdy$
	$\int_0^1 \int_0^y f(x,y)\, dxdy$

I) $\int_0^1 \int_0^x f(x,y)\, dydx$
II) $\int_0^1 \int_0^{1-x} f(x,y)\, dydx$
III) $\int_0^1 \int_x^1 f(x,y)\, dydx$

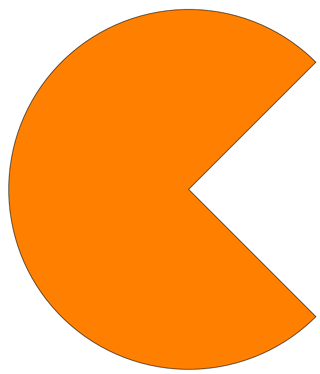
IV) $\int_0^1 \int_0^{x-1} f(x,y)\ dydx$
V) $\int_0^1 \int_{1-x}^1 f(x,y)\ dydx$

Problem 3) (10 points)

The Pac-Man region R is bounded by the lines $y = x, y = -x$ and the unit circle. The number

$$a = \frac{\int \int_R x\ dxdy}{\int \int_R 1\ dxdy}$$

defines the point $C = (a, 0)$ called center of mass of the region. Find it.



Problem 4) (10 points)

Find all the critical points of

$$f(x,y) = x^3 + y^3 - 3x - 12y$$

and indicate whether they are local maxima, local minima or saddle points.

Problem 5) (10 points)

When Ramanujan, an amazing mathematician who was born in India was sick in the hospital and the English mathematician Hardy visited him, Ramanujan asked "whats up?" Hardy answered. "Nothing special. Even the number of the taxi cab was boring: 1729". Ramanujan answered: "No, that is a remarkable number. It is the smallest number, which can be written in two different ways as a sum of two perfect cubes. Indeed $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

a) (5 points) Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = x^3 + y^3 - z^3$ at the point $(9, 10, 12)$.

b) (5 points) Use the technique of linear approximation to estimate $9.001^3+10.02^3-12.001^3$.

Problem 6) (10 points)

a) (5 points) Find the equation $ax + by + cz = d$ for the tangent plane to the level surface

$$f(x, y, z) = x^3 + y^3 - z^3 = 1$$

at the point $(1, 1, 1)$. Note that this is the same Ramanujan function as in the previous problem.

b) (5 points) If we intersect the level surface $f(x, y, z) = 1$ with the plane $z = 2$, we obtain the equation for an implicit curve $x^3 + y^3 = 9$. It is a level curve for the function $g(x, y) = x^3 + y^3$. Find the tangent line to this curve at the point $(1, 2)$.

Problem 7) (10 points)

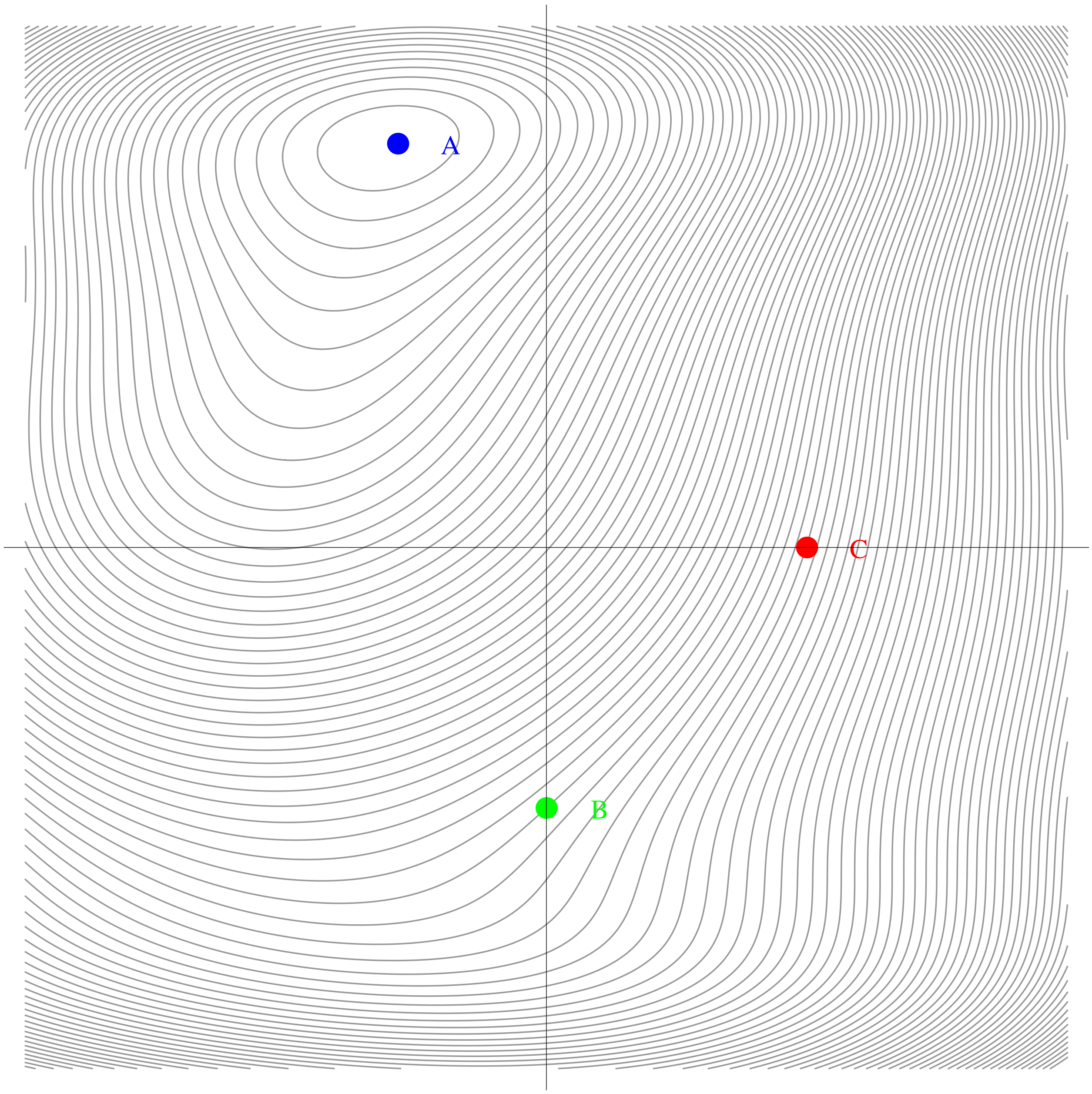
A function $f(x, y)$ of two variables describes the height of a mountain. You don't know the function but you see its level curves. The mountain has its peak at the point A on the picture.

a) (3 points) At which point does f take its maximum under the constraint $x = 0$.

b) (3 points) At which point does f take its maximum under the constraint $y = 0$.

c) (4 points) At which of the points B or C is the length of the gradient vector larger?

Note: As always, you have to give explanations to get full credit. The points in a) and b) are not necessarily marked. Give it up to an accuracy of 1/2. For example, an answer in a) or b) could look like $(0.5, 1.5)$.



Problem 8) (10 points)

Olivers great-grand-dad Emil Frech Hoch (1874-1947) founded the car company "Frech-Hoch" in Switzerland. The company produces cars and trucks. The company still exists today and produces specialized vehicles.

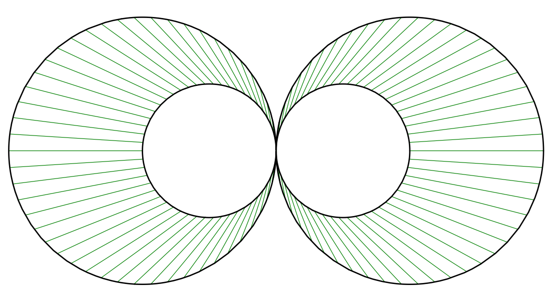
Assume the revenue of the company is $f(x, y) = x^2 + 2y^2$, where x is the number of cars and y is the number of trucks produced per year. The production is constrained by the amount of steel available. Trucks need twice as much steel leading to $g(x, y) = x + 2y = 1$. Use the Lagrange multiplier method to find the optimal production rate.

Problem 9) (10 points)

Find the area of the region in the plane given in polar coordinates by

$$\{(r, \theta) \mid |\cos(\theta)| \le r \le 2|\cos(\theta)|, 0 \le \theta < 2\pi \} .$$

The region is the shaded part in the figure.



Problem 10) (10 points)

What is the shape of the triangle with angles α, β, γ for which

$$f(\alpha, \beta, \gamma) = \log\left(\sin(\alpha)\sin(\beta)\sin(\gamma)\right)$$

is maximal?