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Cairaut: $f_{xy} = f_{yx}$ Proof: take $h \rightarrow 0$ limit of
 $f_{xy} \sim h^2[(f(x+h, y+h) - f(x, y+h)) - (f(x+h, y) - f(x, y))]$
 $f_{yx} \sim h^2[(f(x+h, y+h) - f(x+h, y)) - (f(x, y+h) - f(x, y))]$



ODE: equation for function f involving derivatives

PDE: equation for function f involving partial

derivatives. Example: $f_x = f_{yy}f + f^2$

Heat equation: $u_t = u_{xx}$

Wave equation: $u_{tt} = u_{xx}$

Laplace equation $u_{xx} + u_{yy} = 0$

Transport equation $u_t = u_x$

Burgers equation $u_t + uu_x = u_{xx}$

Gradients are perpendicular to level sets.

Proof: $\vec{r}'(t)$ on $f = c$ satisfies

$$0 = d/dt f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}'(t)$$

showing that ∇f is orthogonal



Directional derivative maximizes in gradient direction. Proof:

$$D_{\vec{v}}f = |\nabla f \cdot \vec{v}| = |\nabla f| \cos(a)$$

For $\vec{w} = \nabla f / |\nabla f|$,

$$D_{\vec{w}}f = |\nabla f|$$

Second derivative test: discriminant

$D = f_{xx}f_{yy} - f_{xy}^2$ and $A = f_{xx}$ determine:

$D > 0, A > 0 \Rightarrow \min, D > 0, A < 0 \Rightarrow \max,$

$D < 0 \Rightarrow \text{saddle}, D = 0 \text{ not know.}$

Lagrange:

$$f_x = Lg_x$$

$$f_y = Lg_y$$

$$g(x, y) = c$$

Proof: $\nabla(f)$ and $\nabla(g)$ are parallel. Else

moving on $g = c$ crosses level curves.



Chain rule $\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}'(t)$. Implicit differentiation $f(x, g(x)) = 0$ implies $g'(x) = -f_x/f_y$.

Tangent plane at P : find $\nabla f = [a, b, c]$ and plane
 $ax + by + cz = d$, (get d by plugging in P).
 $\nabla f = [a, b]$ gives tangent line $ax + by = d$.



Estimate $f(3.001, 4.9999)$ by computing the gradient $[a, b]$ of
 f at $(3, 5)$ and get
 $L(3.001, 4.9999) = f(3, 5) + a \cdot 0.001 - b \cdot 0.0001$.



“Bottom to top” integration on $[a, b]$ on x-axis
“Left to right” integration on $[c, d]$ on y-axis

Double integral $\int \int_R f(x, y) dx dy$ interpretation:
signed volume under the graph of f . It is volume if $f \geq 0$.

Fubini: for rectangular regions only:
 $\int_a^b \int_c^d f(x, y) dx dy = \int_c^d \int_a^b f(x, y) dy dx$

Surface area of a parametrized surface $\vec{r}(u, v)$, defined for a
region R is $\int \int_R |\vec{r}_u \times \vec{r}_v| du dv$.

Polar integration:
include an integration factor r .
Proof: $\vec{r}(s, t) = [r \cos(t), r \sin(t), 0]$, $|\vec{r}_r \times \vec{r}_t| = r$.



By parts: $\int u dv = uv - \int v du$
Proof: integrate $uv' + vu' = (uv)'$
Example: $\int x \cos(x) dx = x \sin(x) - \int 1 \cdot \sin(x) dx = x \sin(x) + \cos(x) + C$



Substitution: Example:
 $\int x^4 \exp(x^5) dx$
 $u = x^5, du = 5x^4 dx \int \exp(u)/5 du = \exp(x^5)/5 + C$

Tips for double integrals: make picture, consider other co-
ordinates and change order of integration.

Helpful identities:
 $\cos^2(t) + \sin^2(t) = 1$
 $\cos^2(t) = (1 + \cos(2t))/2$
 $\sin^2(t) = (1 - \cos(2t))/2$
 $\sin(t) \cos(t) = \sin(2t)/2$



$\int x^n dx = x^{n+1}/(n+1)$
 $\int \exp(ax) dx = \exp(ax)/a$
 $\int \cos(ax) = \sin(ax)/a$
 $\int \sin(ax) = -\cos(ax)/a$
 $\int dx/x = \log(x)$
 $\int dx/(1+x^2) = \arctan(x)$



Know integration by parts, substitution, basic integrals!