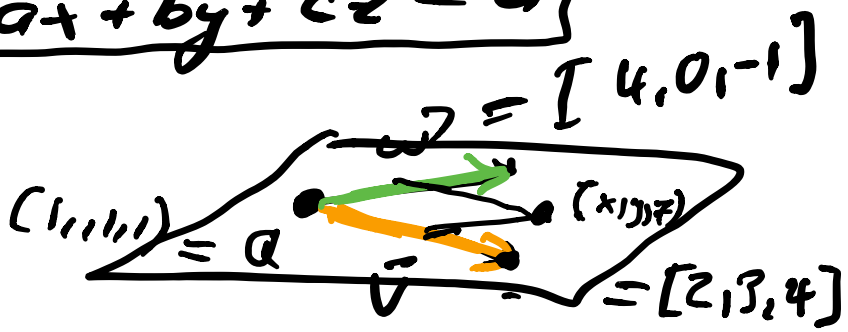
Unit 4

Lines and distances

① Planes

$$ax + by + cz = d$$



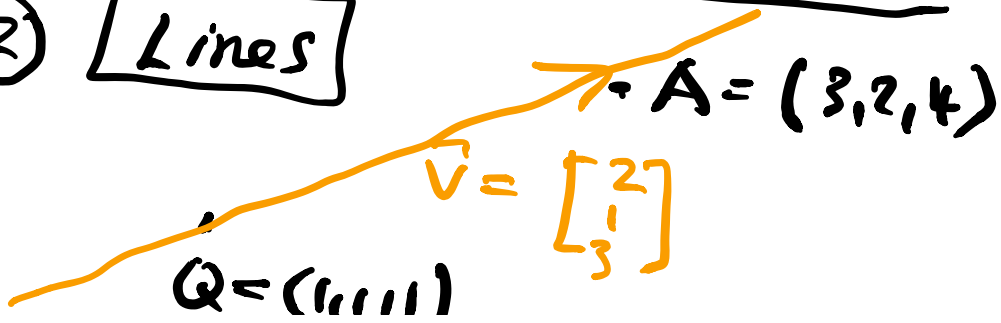
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + t\begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 2s + 4t \\ 1 + 3s + 0t \\ 1 + 4s - t \end{bmatrix}$$

parameterization of the plane

---

② Lines



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

parametrization of line

$$\begin{aligned} x &= 1 + 2t \\ y &= 1 + t \\ z &= 1 + 3t \end{aligned}$$

plane

Solve for t

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{3}$$

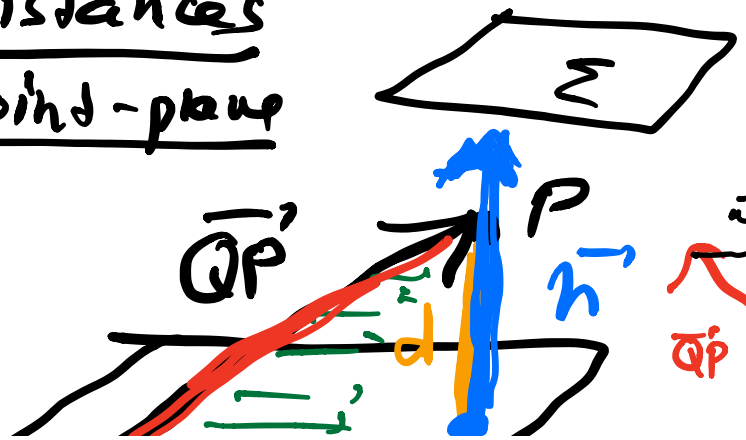
Symmetric equations

plane

P

(3) Distances

point-plane



$$P_{\vec{n}}\vec{v} = \frac{\vec{n}\cdot\vec{v}}{|\vec{n}|^2}\vec{n} = \frac{\vec{n}\cdot\vec{v}}{|\vec{n}|}\left(\frac{\vec{n}}{|\vec{n}|}\right)$$

length

$$d = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|}$$

onto $\vec{n}$

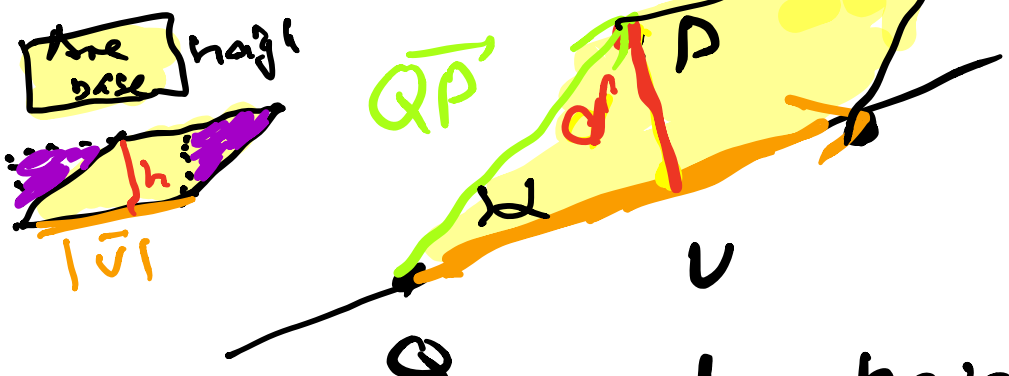
length of projection vector

$$= \frac{|\overrightarrow{QP} \cdot (\vec{v} \times \vec{\omega})|}{|\vec{v} \times \vec{\omega}|}$$

$$= \frac{\text{Volume}}{\text{Area}} = \text{height}$$

---

(4) Point - line distance



$|v|$ = base length

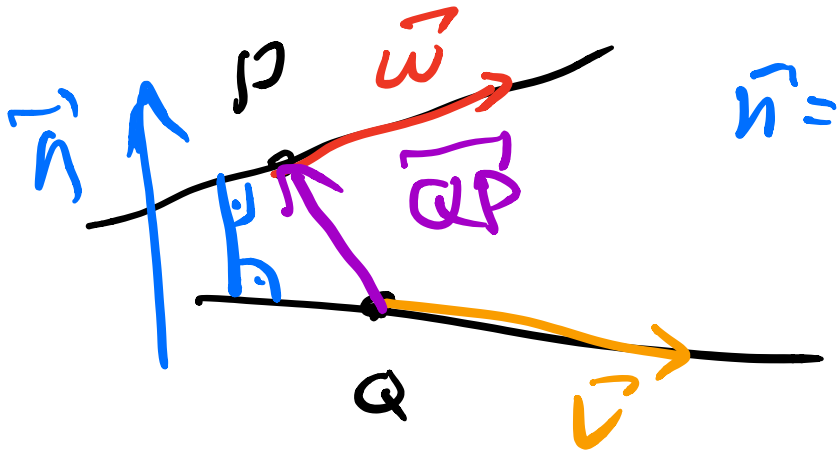
d = height

$= \frac{\text{Area}}{}$

Area $= |\vec{v} \times \vec{QP}|$ $\overset{bas}{=} \dfrac{|\vec{v} \times \vec{QP}|}{|v|}$

---

(5) Line – Line distance



$\vec{n} = \vec{v} \times \vec{w}$

$d =$ Projection of $\vec{QP}$ onto $\vec{n}$

$$= \left| \frac{\vec{QP} \cdot \vec{n}}{|n|} \right| = \frac{|\vec{QP} \cdot (\vec{v} \times \vec{w})|}{|\vec{v} \times \vec{w}|}$$

geometrically:

$$\frac{\text{Volume}}{\text{Area}}$$