

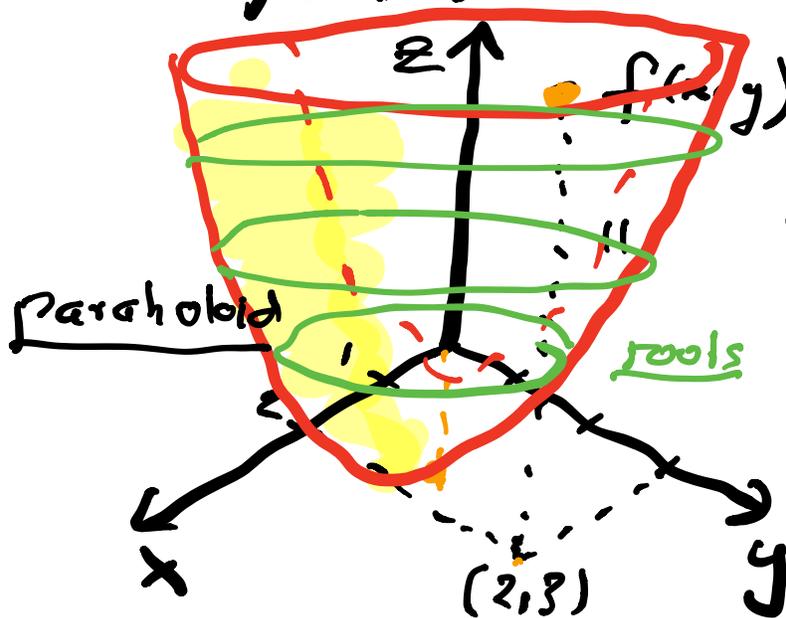
Unit 5 Functions

① Functions of 2 variables

$$f(x,y) = x^2 + y^2 - 2$$

$$f(2,3) = 11$$

rotationally symmetric

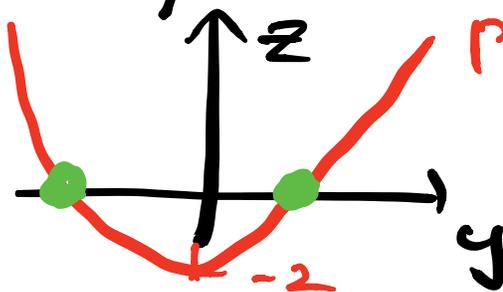


$\{(x,y,z) \in \mathbb{R}^3, z = f(x,y)\}$
graph of f

Traces: $x = 0$: $y-z$ -coordinate plane.

$$z = y^2 - 2$$

domain \mathbb{R}^2 -plane
range $[-2, \infty)$

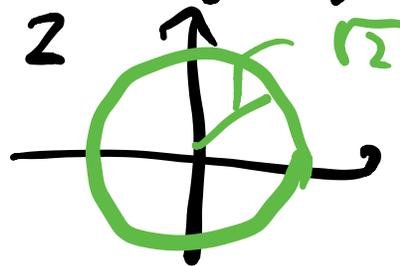


$y = 0:$
 $z = x^2 - 2$

$$z=0$$

x - y -plane
roots of f

$$x^2 + y^2 = 2$$



generalized
trajectories:

$$z=c$$

$$z=1$$

$$x^2 + y^2 - 2 = 1$$

$$x^2 + y^2 = 3$$

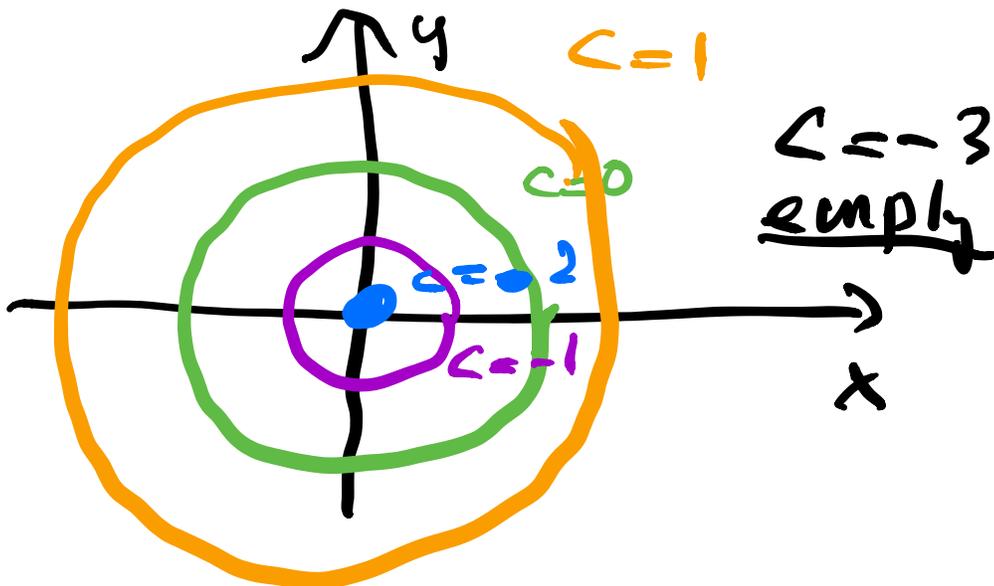


Level curves:

$$z=c$$

\approx contour curves

contour map



Applications:

Weather:

Functions:
isotherms, isobars

$T(x,y)$ temperature
 $P(x,y)$ pressure

Altitude

Function:

$f(x,y) = \text{height}$

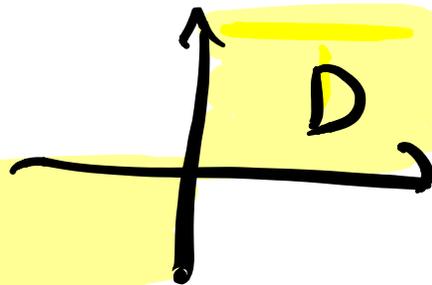
$f(x,y) = c$, iso heights

domain:

D

$\{ \text{All } (x,y),$
 $f(x,y) \text{ is}$
 $\text{defined} \}$

$f(x,y) = \sqrt{xy}$
Range: $[0, \infty)$

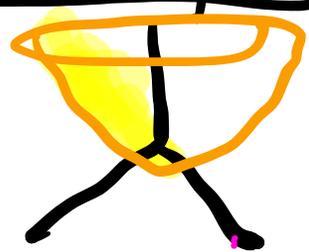


range:

all function
values which
are reached.

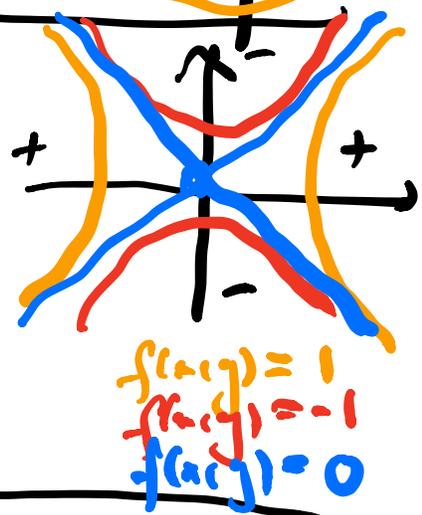
(2) Examples

a) $f(x,y) = x^2 + y^2$
 elliptic
 paraboloid



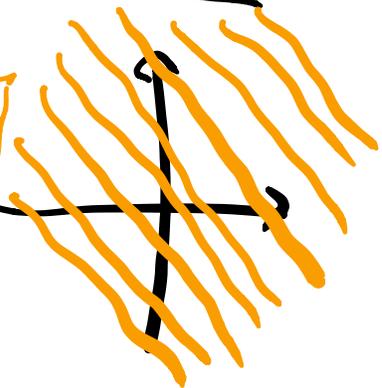
b) $f(x,y) = x^2 - y^2$

$\nabla f = 0$
 $\nabla^2 f = x^2$
 $x = 0$
 $\nabla^2 f = -y^2$
 $x^2 - y^2 = 0$

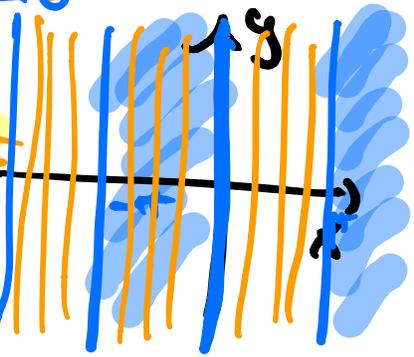


c) $f(x,y) = 6 - x - 2y = z$

in krepels



d) $f(x,y) = \sin x \approx 0$ roots



③ Functions of 3 variables

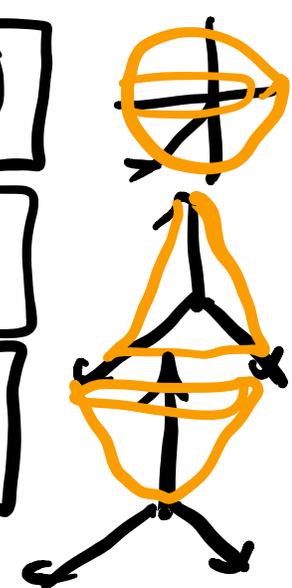
We can no more draw the graph? we would need hyperspace.

Contours $f(x,y,z) = c$

a) $x^2 + y^2 + z^2 = 1$ sphere

b) $2x + 3y + z = 6$ plane

c) $z - x^2 - y^2 = 0$ paraboloid



d) Homework :

$$|x| + |y| + |z| = 1$$

Octahedron





④ Quadratics

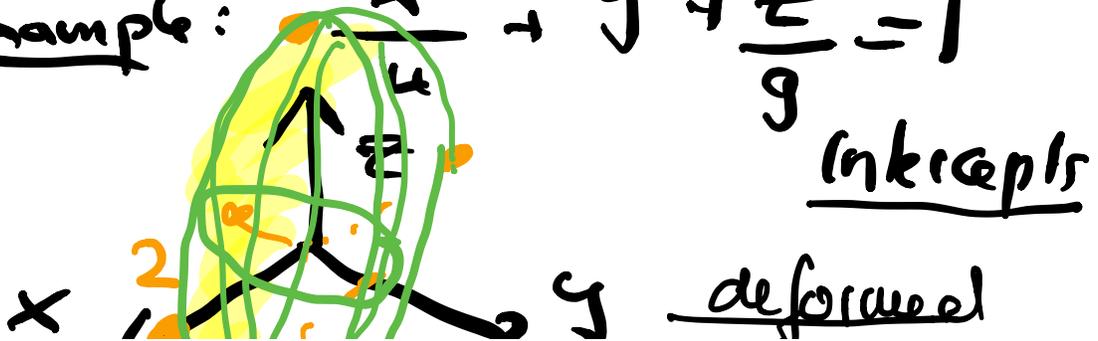
quadratic function

- Ⓐ Ellipsoids
- Ⓑ Parabolas
- Ⓒ Hyperboids
- Ⓓ Special

Ⓐ Ellipsoids ← sphere general.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Example: $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$





spheres

(B) Paraboloids

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$



elliptic paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

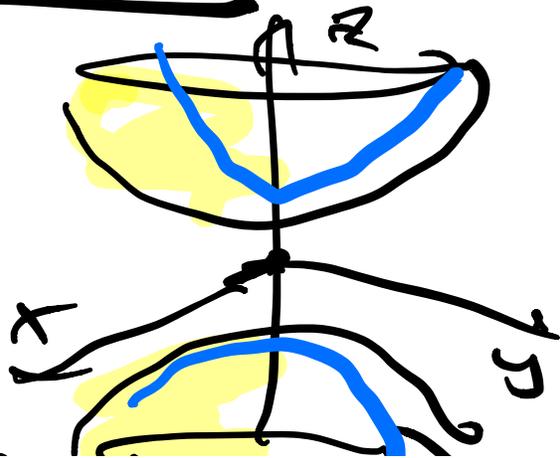
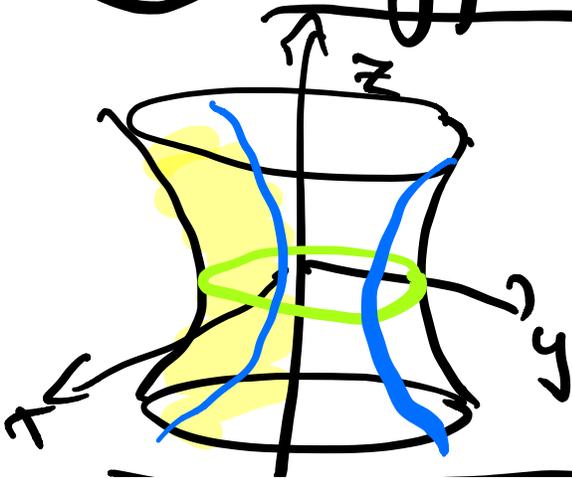


hyperbolic paraboloid

they are all graphs

$$z = f(x, y) \quad \leftarrow \text{graph}$$

(C) Hyperboloid



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

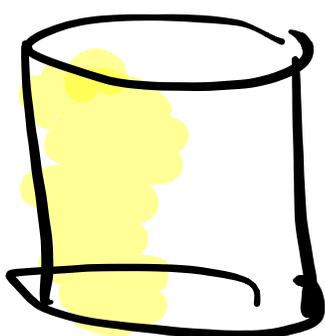
$z=0$
 ~~$z=0$~~
 ~~$y^2 - z^2 = 1$~~
 hyperbola

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

$z=0$ does not give any things

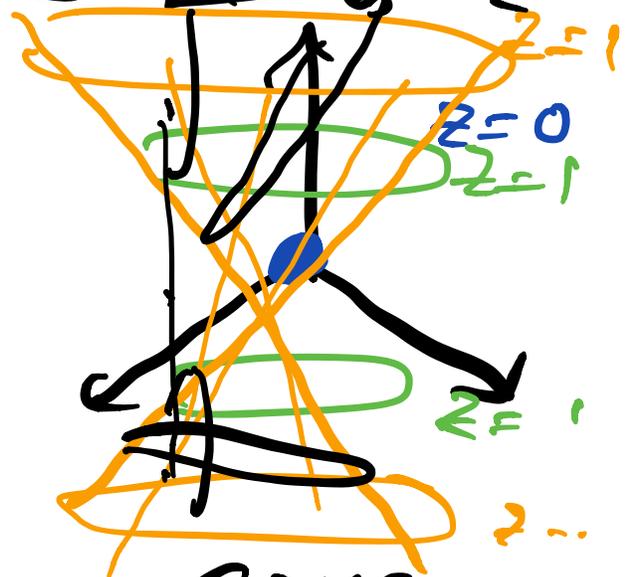
D) Special

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



Cone

Conic Sections

hyperbola, parabola, ellipses.

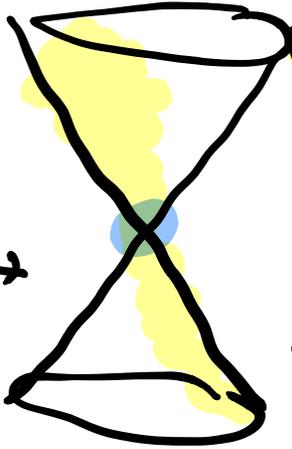
1. Reformation

$$x^2 + y^2 - z^2 = c$$

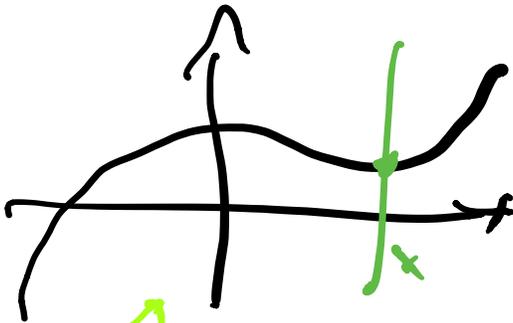
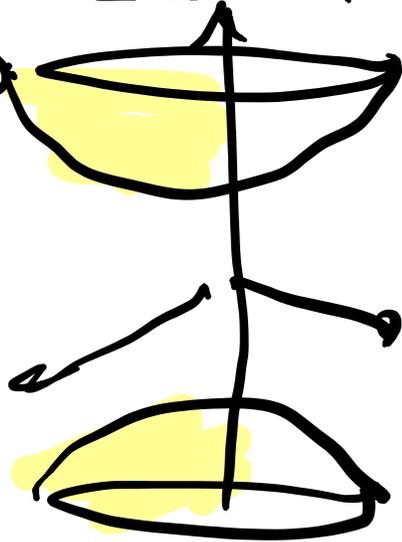
$$c = 1$$



$$c = 0$$

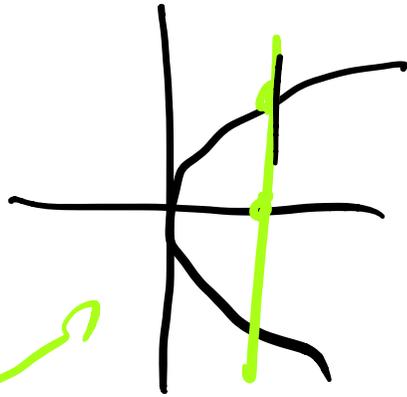


$$c = -1$$



graph

$$y - \sin x - 1 = 0$$



Not a graph

$$y = \pm \sqrt{x}$$

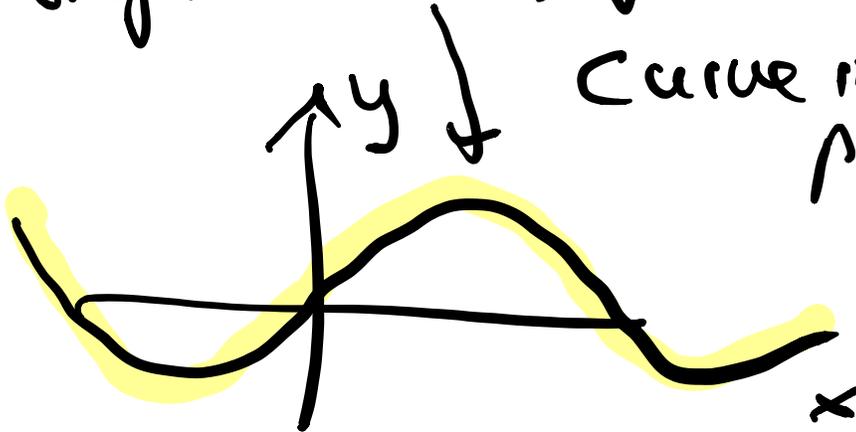
$$y^2 = x$$

$$x - y^2 = 0$$

~~graph~~
single vari.

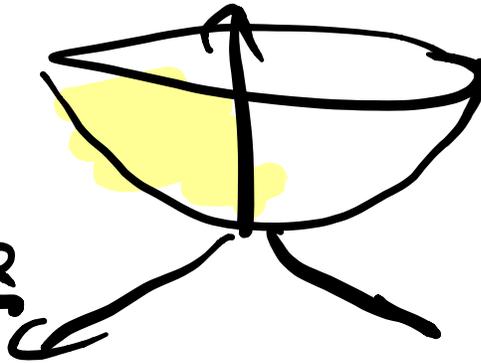
$$y = f(x)$$

Curve in 2D
plane



~~graph~~
2 vari

~~Surface~~



$$z = f(x, y)$$