$\boxed{\text{unit 9}}$

$\partial$ partial
$\delta$ delta

## (1) Partial derivatives

$$f(x,y) = \sin(x)\cdot y^2$$

$$f_x = \frac{\partial}{\partial x} f(x,y) = \cos x\ y^2$$

$$f_y = \frac{\partial}{\partial y} f(x,y) = 2\sin x\ y$$

$$f_{xx} = \frac{\partial^2}{\partial x^2} f(x,y) = -\sin x\ y^2$$

$$f_{yy} = \frac{\partial^2}{\partial y^2} f(x,y) = 2\sin x$$

$$f_{xy} = \frac{\partial}{\partial y}\frac{\partial}{\partial x} f(x,y) = \boxed{2y\cos x}$$

$$f_{yx} = \frac{\partial}{\partial x}\frac{\partial}{\partial y} f(x,y) = \boxed{2\cos x\ y}$$

## (2) $\boxed{\text{Clairaut}}$

$h \approx 10^{-34}$ cm

$$\boxed{f_{xy} = f_{yx}}$$

f is twice cont. differen.

$$h f_x = [f(x+h,y) - f(x,y)]$$

$$\boxed{h^2 f_{xy}} = [f(x+h,y+h) - f(x,y+h)] - [f(x+h,y) - f(x,y)]$$

$$h f_y = [f(x,y+h) - f(x,y)]$$

$$\boxed{h^2 f_{yx}} = [f(x+h,y+h) - f(x+h,y)] - [f(x,y+h) - f(x,y)]$$

Now take the limit $h \to 0$.

Quantum calculus

calculus without limit.

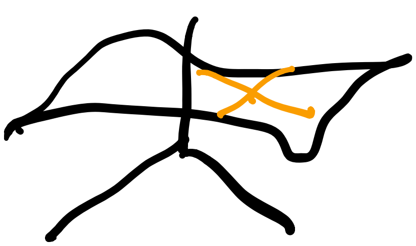
Atiyah

finitists

Browe

no infinity

no infinites



Slopes

③ $\boxed{\text{PDE's}}$

Partial differential equation

ODE only one variable

$$f(t, x)$$

funct, time, space

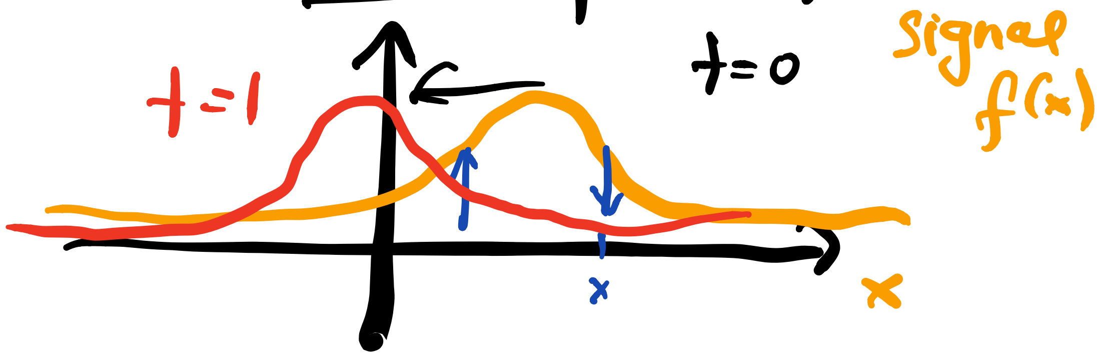
"knit" equation

$$\boxed{f_{ttt} = f_{xxt} + f_{xt}}$$

what functions f satisfies this?

a) $\boxed{f_t = f_x}$

Transport equation



$g(x) = f(0,x)$

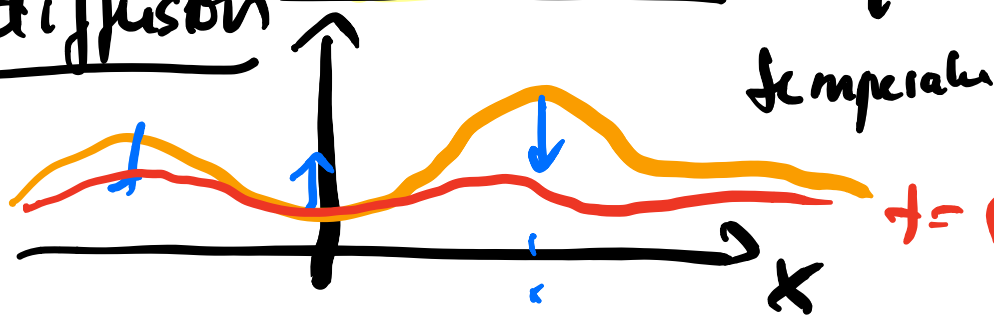
initial signal.

$f(t,x) = g(t+x)$

solves transport equ.

b) $\boxed{f_t = f_{xx}}$ heat equation

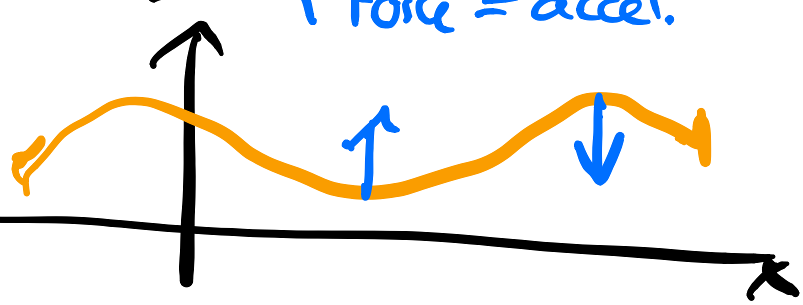
diffusion



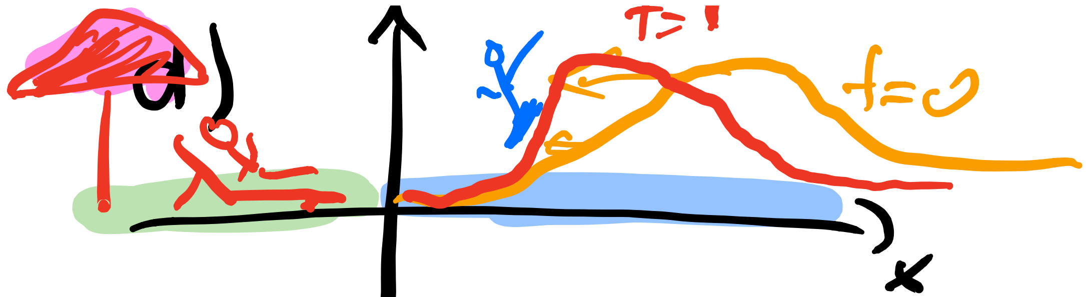

Concave du $f_{xx} < 0$

c) $\boxed{f_{tt} = f_{xx}}$

Force = accel.



d)




d) $\boxed{f_t = f f_x}$ $(+ f_{xx})$

$$f_t = \overset{\text{speed}}{c} f_x$$

is solved by

$$g(ct + x)$$

$$g(x) = f(0,x)$$

Here: speed $c$ is actually $f$ which is the height

e) $$f_{tt} + f_{xx} = 0$$

**Laplace equal**

$$\Delta f = f_{xx} + f_{yy} = 0$$

$$\boxed{f_t = i f_{xx}} + V(x) f$$

Schroedinger equation

$$\boxed{i f_t = -\Delta f + V(x) f}$$

explaines all periodic system of elements