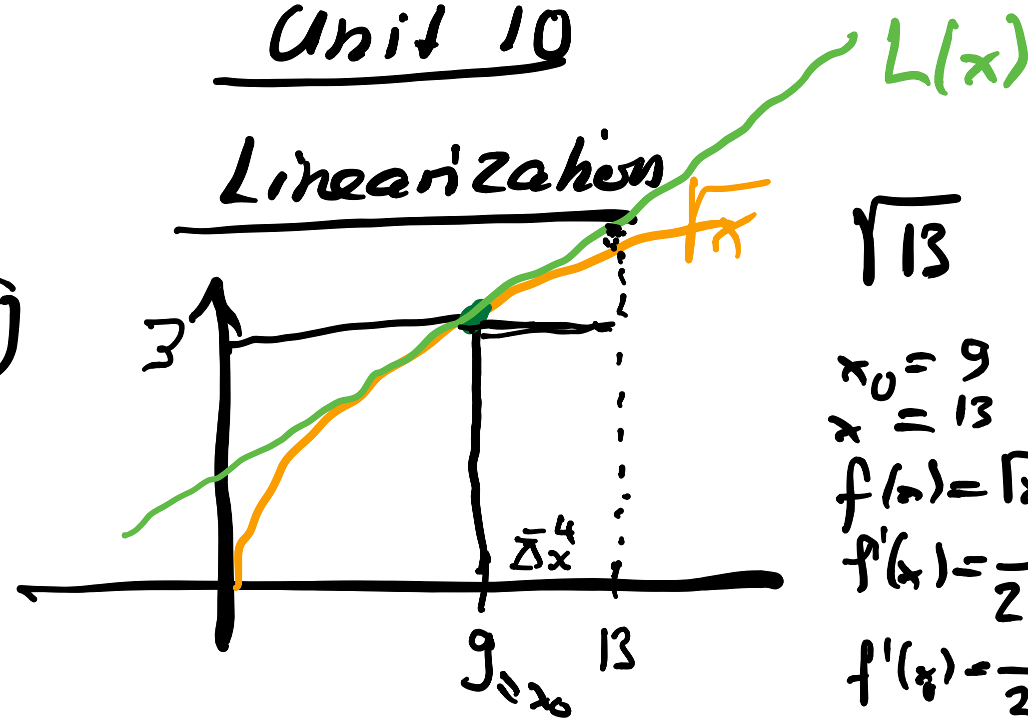
Unit 10

Linearization

(1)



$\sqrt{13}$

$x_0 = 9$

$x = 13$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(9) = \frac{1}{2\cdot 3}$

$$\boxed{L(x) = f(x_0) + f'(x_0)(x - x_0)}$$

Linearization of f at x_0

$$L(x) = 3 + \frac{1}{6}\cdot(x-9)$$

graph of L is the tangent line

$$L(13) = 3 + \frac{1}{6}\cdot 4 = 3 + \frac{4}{6}$$

$$= 3.66666$$

3.60555

Example: $\sqrt{125} = 11.1803$

$121 = x_0$

Estimate: $11 + \frac{4}{2\cdot 11} = 11 + \frac{4}{22}$

11.1818

Example: Cube root.

$\sqrt[3]{973} = 10 - \frac{27}{3\cdot 100}$

$= 9.90918$

$= 9.91$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3(x^{1/3})^2}$

$f(x) = x^{1/5}$

$f'(x) = \frac{1}{5} x^{-4/5}$

$= \frac{1}{5(x^{1/5})^4}$

② Higher dimension

numbers

$$\boxed{\begin{aligned} L(x,y) = f(x_0,y_0) \\ + f_x(x_0,y_0)(x-x_0) \\ + f_y(x_0,y_0)(y-y_0) \end{aligned}}$$

Linearization of $f(x,y)$ at (x_0,y_0)

Example: $\boxed{\sqrt[3]{10.001^2 \cdot 9.99}}$

$= \sqrt[3]{x^2 \cdot y} = f(x,y) = x^{2/3} y^{1/3}$

$(x_0,y_0) = (10,10)$

$f(x_0,y_0) = 10$

$f_x(x,y) = \frac{2}{3} x^{-1/3} y^{1/3}$

$$f_y(x,y) = \frac{1}{3} x^{2/3} y^{-2/3}$$

$$f_x(10,10) = \frac{2}{3}$$

$$f_y(10,10) = \frac{1}{3}$$

$$\boxed{L(x,y) = 10 + \frac{2}{3}(x-10) + \frac{1}{3}(y-10)}$$

$$10 + \frac{2}{3}\, 0.001 + \frac{1}{3}(-0.01)$$

$x = 10.001$

$y = 9.99$

$= 9.99733$

Real value: 9.99733

(3) **Higher dimensions**

$$f(x,y,z)$$

$$L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0)$$

New notation!

$$\nabla f(x,y) = [f_x, f_y]$$

$$\nabla f(x,y,z) = [f_x, f_y, f_z]$$

gradient

Nabla

$$\boxed{L(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0)\cdot(\vec{x}-\vec{x}_0)}$$

$$[f_x, f_y]\cdot[x-x_0, y-y_0]$$
$$= f_x(x-x_0)+f_y(y-y_0)$$

$$\vec{X} = [x, y, z]$$

$$\vec{x} = [x, y]$$

$$\vec{x}_0 = [x_0, y_0]$$

In 1 dim

$$\nabla f(x) = f'(x)$$