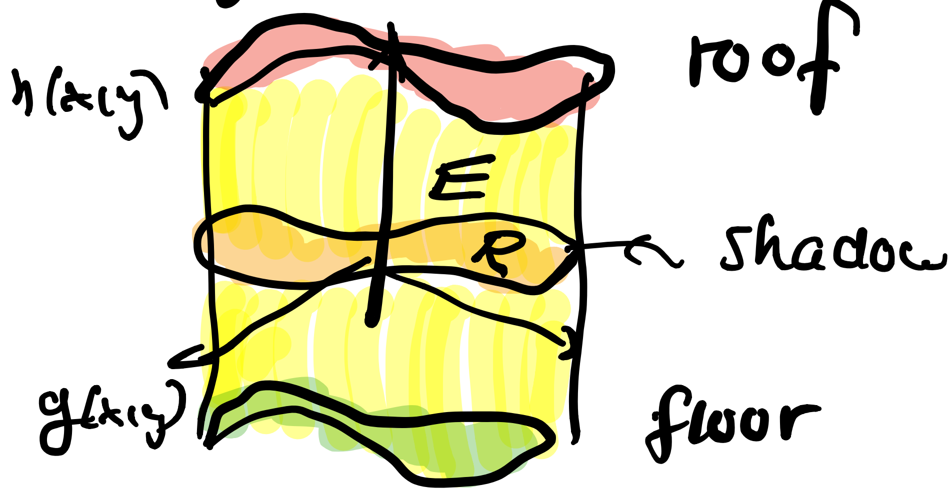
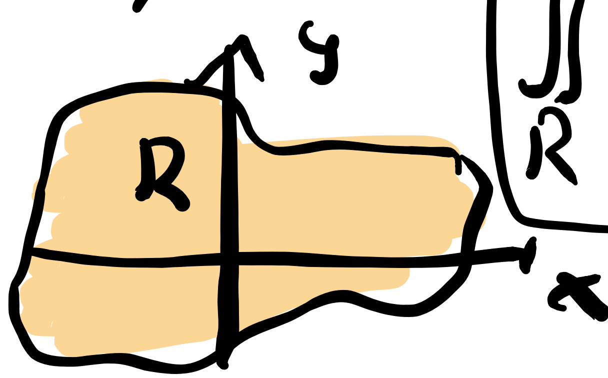
Unit 17

1) Triple integrals

reduce to double integrals



E solid usually sandwiched between two functions.



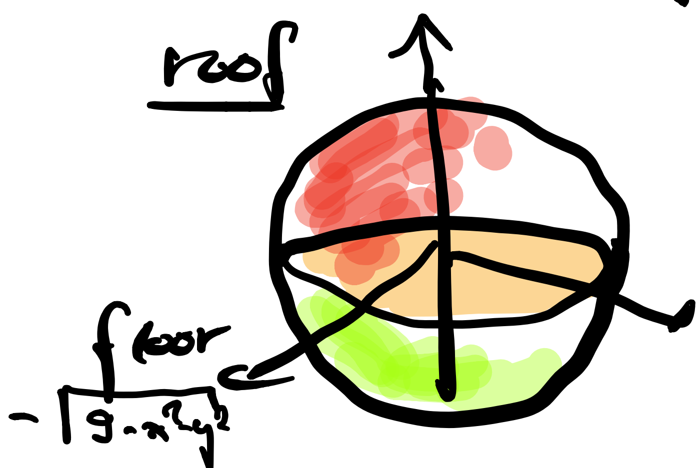
$$\iint_R \int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\,dA$$

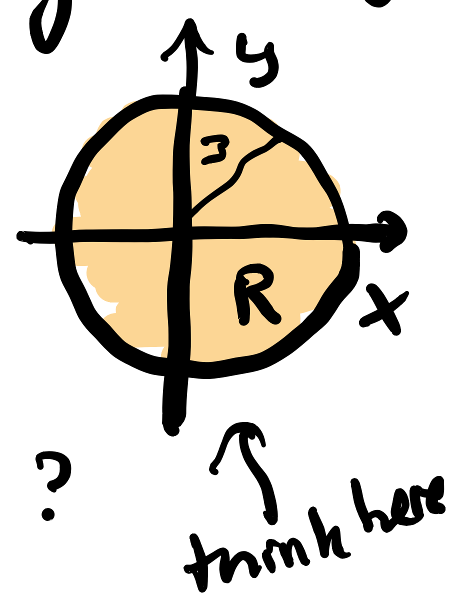
To compute volumes we take $f(x,y,z)=1$

$$\boxed{\iiint_E 1\, dV = \underline{\text{volume}}}$$

sphere

② **Example:** Sphere $x^2+y^2+z^2 \le 9$ ← Solve for





What is the roof? $\sqrt{9-x^2-y^2}$

$$\iint_R \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} 1 \, dz\, dA$$

floor

reduce to double integral!

$$\boxed{\iint_R 2\sqrt{\underbrace{9-x^2-y^2}_{9-r^2}}\,dx\,dy}$$

Review mode: "flatland"

Switch to polar!

integration factor

$$\int_0^{2\pi}\int_0^{3} 2\sqrt{9-r^2}\;\boxed{r}\;dr\,d\theta$$

substitute $u = 9-r^2$
$du = -2r\,dr$

$$\int_0^{2\pi} -(9-r^2)^{3/2}\cdot\frac{2}{3}\Big|_0^3\,d\theta$$

$$\int_0^{2\pi} 0 + 3^3\cdot\frac{2}{3}\,d\theta = \boxed{\frac{4\pi}{3}3^3}$$

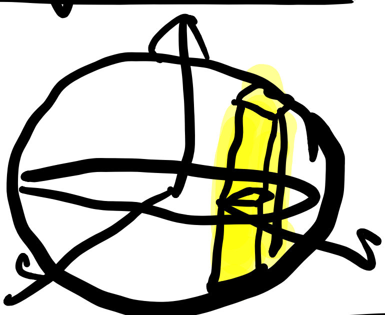
$\longrightarrow$ much easier in unit 18

Volume of sphere Archimed.

Single variable

Burger method

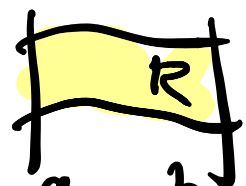
$$\int_{-3}^{3} A(z)\,dz = \text{Vol}$$



Fries method.

→ reduce to double integral

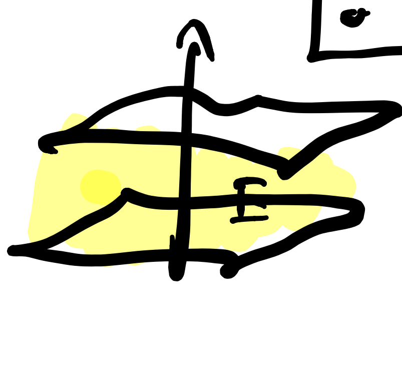
Question:



$$\iint T\,dx\,dy \longleftarrow \int_a^b \int_{g(x)}^{h(x)} T\,dy\,dx$$

$f(x,y)$

Area $= \boxed{\int_a^b h(x) - g(x)\,dx}$



$$\iiint_E f(x,y,z)\,dV$$

density.

If $f(x,y,z)$ is not 1.

then this a different quanti

$$\iiint_E f(x,y,z)\, dV$$

density

= Mass

$$\iiint_E x^2+y^2\, dV$$

Moment of inertia

In probability theory

prob. den density

$$\iiint_E f(x,y,z)\, dx\,dy\,dz$$

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③ An exam problem

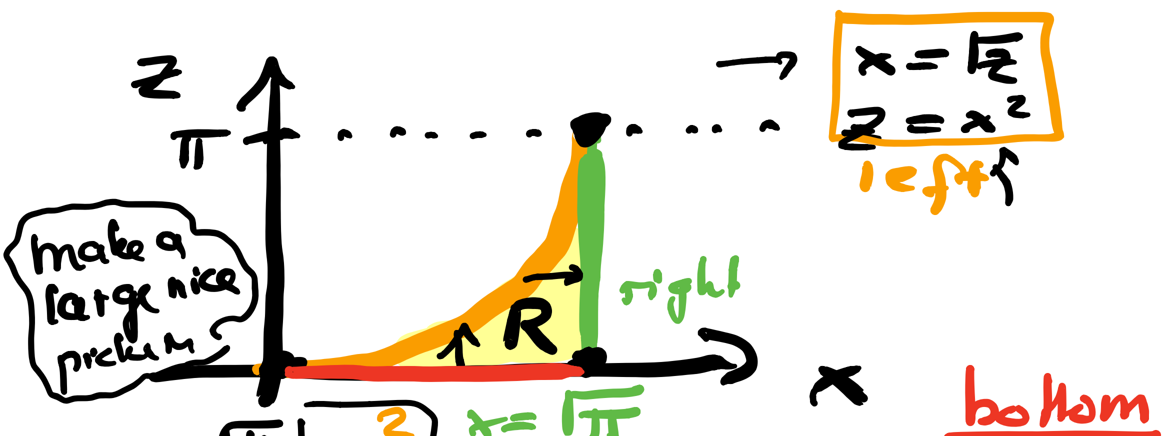
$$\boxed{\int_0^{\pi}\int_{\sqrt{z}}^{\sqrt{\pi}}\int_0^{x} \sin(xy)\, dy\, dx\, dz}$$

$$\int_0^{\pi}\int_{\sqrt{z}}^{\sqrt{\pi}} -\frac{\cos(xy)}{x}\Bigg|_0^{x} dx\, dz$$

$$\int_0^{\pi}\int_{\sqrt{z}}^{\sqrt{\pi}} \frac{1-\cos x^2}{x}\,dx\,dz$$

Review mode : try to solve this double integral !

we are stuck! Remember what to do here :
change the order of integration
To do that, you **must** make a picture



$$\int_0^{\sqrt{\pi}}\int_0^{x^2} \frac{1-\cos x^2}{x}\,dz\,dx$$

top was "left"

$$\int_0^{\sqrt{\pi}} x^2\left(\frac{1-\cos x^2}{x}\right)dx$$

Sub.

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int_0^{\sqrt{\pi}} x - x\cos(x^2)\,dx$$

$$= \left.\frac{x^2}{2} - \frac{\sin x^2}{2}\right|_0^{\sqrt{\pi}}$$

$$= \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$$