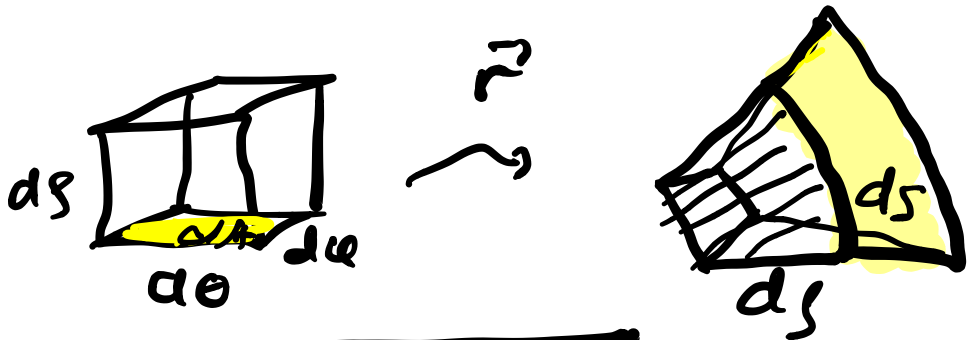
Unit 18

Spherical integration



$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$\vec{r}(\theta, \phi, \rho) = [x, y, z]$

Remember:

Integration factor

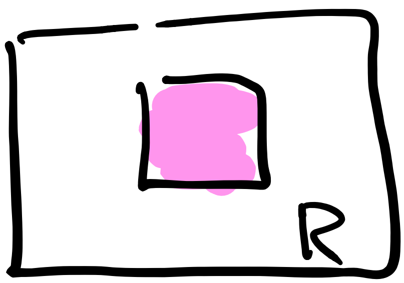
$$|\vec{r}_\theta \times \vec{r}_\phi| = \rho^2 \sin\phi$$

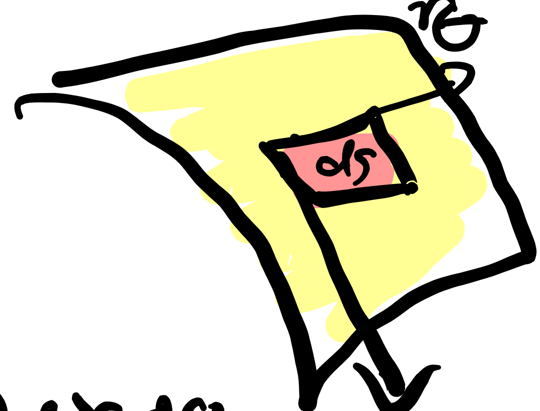
The volume

$d\phi\, d\theta\, d\rho \to \rho^2 \sin\phi\, d\phi\, d\theta\, d\rho$

$\rho^2 \sin\varphi$ is the integration factor to be included, when going into spherical coordinates

It plays the role of r in polar coordinates.





r_φ

$dS = \boxed{\rho^2 \sin\varphi} \, d\varphi \, d\theta$

surface area element.

Surface area: $\iint_R |r_\varphi \times r_\theta| \, d\varphi \, d\theta$

② Volume of Sphere

$$\iiint_E 1 \, dV$$

<u>Volume</u>

$$\int_0^{2\pi} \int_0^{\pi} \boxed{\int_0^3 1 \cdot \rho^2 \sin\varphi \, d\rho} \, d\varphi \, d\theta$$

$$\frac{\rho^3}{3}\Big|_0^3 \cdot \sin\varphi$$

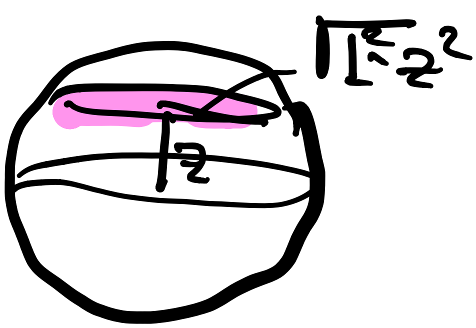
$$\int_0^{2\pi} \int_0^{\pi} \left(\frac{3^3}{3} - 0\right) \sin\varphi \, d\varphi \, d\theta$$

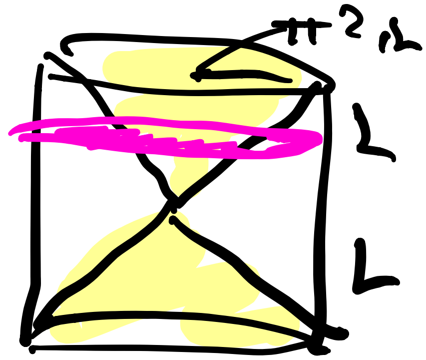
$$= 9\int_0^{2\pi}\left[\int_0^{\pi}\sin\varphi\, d\varphi\right] d\theta$$

$$-\cos\varphi\Big|_0^{\pi} = 2$$

$$= 9 \cdot 2 \cdot 2\pi = 36\pi$$

$3 \to L$ $\quad \boxed{\frac{4\pi L^3}{3}}$ $\quad$ **Archimedes**





$$A(z) = (L^2 - z^2)\pi$$

$$\int_{-L}^{L} (L^2 - z^2)\pi \, dz$$

$$L^2\pi - z^2\pi$$

Vol sphere =
Vol cylinder

$$2L \cdot \pi^2 L^2 - \frac{2L\pi^2 L^2}{3}$$

vol.cyl. — vol con

$$= \frac{4\pi L^3}{3}$$

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③ **Moment of inertia**

$$I = \iiint_E (x^2+y^2)\, dV$$

moment of inertia

Energy of spinning object:

$$I \cdot \frac{\omega^2}{2}$$

ω is angular velocity.

RPM rounds per minute

5400RPM Laptop HD

For the sphere:

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{L} \boxed{\rho^2 \sin^2\varphi}\, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$x^2+y^2 = \rho^2 \sin^2\varphi$$

$$\int_0^{2\pi}\int_0^{\pi} \sin^3\varphi \boxed{\int_0^{L} \rho^4 \, d\rho} \, d\varphi \, d\theta$$

$$\frac{L^5}{5}$$

$$2\pi \frac{L^5}{5} \int_0^{\pi} \sin^3\varphi \, d\varphi$$

$$\sin^3\varphi = \sin^2\varphi \quad \sin\varphi$$

$$= (1-\cos^2\varphi)\sin\varphi$$

$$= \sin\varphi - \cos^2\varphi \sin\varphi$$

$$\int_0^{\pi} \sin^3\varphi \, d\varphi = -\cos\varphi + \frac{\cos^3\varphi}{3}\Big|_0^{\pi}$$

$$= 2 - \frac{2}{3}$$

$$= \frac{4}{3}$$

$$2\pi \frac{L^5}{5} \cdot \frac{4}{3} = \boxed{\frac{8\pi L^5}{15}}$$

<u>Earth</u>: $L = 6000 km$
$= 6 \cdot 10^6 m$

$$\omega = \frac{1}{1 day} = \frac{1}{24h}$$
$$= \frac{1}{24 \cdot 3600} s^{-1}$$

angulo velo.

Energy of Earth:

$$\frac{1}{24^2 \cdot 3600^2} \frac{1}{2} \frac{8\pi}{15} \left(6 \cdot 10^6\right)^5 \; S$$

Joules.

1 Apple : 60 Kkal

240 kJoule

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xkcd

what if

Explain ..

Intermission