$\boxed{\text{Unit 19}}$

$\vec{F}(2,1) = [1,2]$

$(2,1)$ velocity field

1)

a)

rubber duck

$$\vec{F}(x,y) = [P(x,y), Q(x,y)]$$

VectorPlot
StreamPlot

$$= [-y, x]$$

Vector field in $\mathbb{R}^2$

Flow line:

$$\boxed{r'(t) = \vec{F}(r(t))}$$

$$\vec{r}(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\vec{r}\,'(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$

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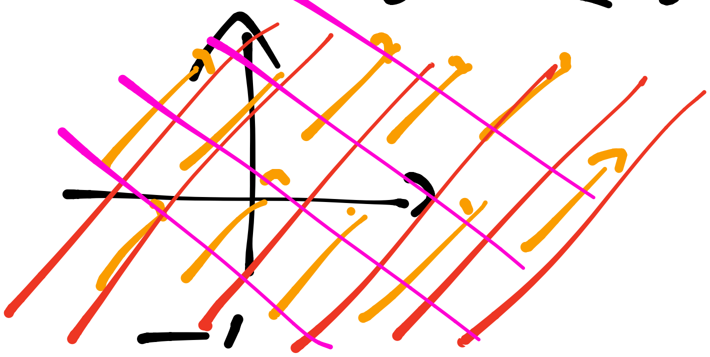
b) $\vec{F}(x,y) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \nabla f$



no equilibri

$f = 2x + 3y$

$\vec{F}(x,y) = [0,0]$

equilibrium

$f_x \cdot f_y$

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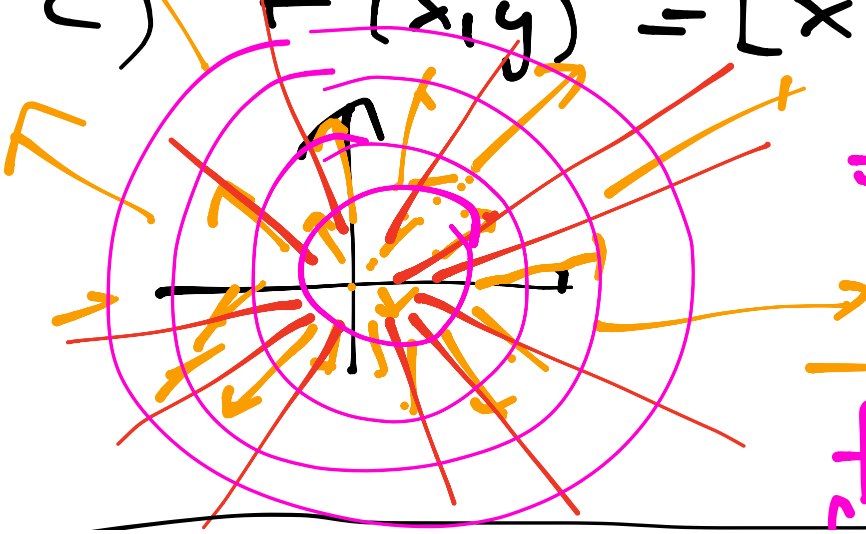
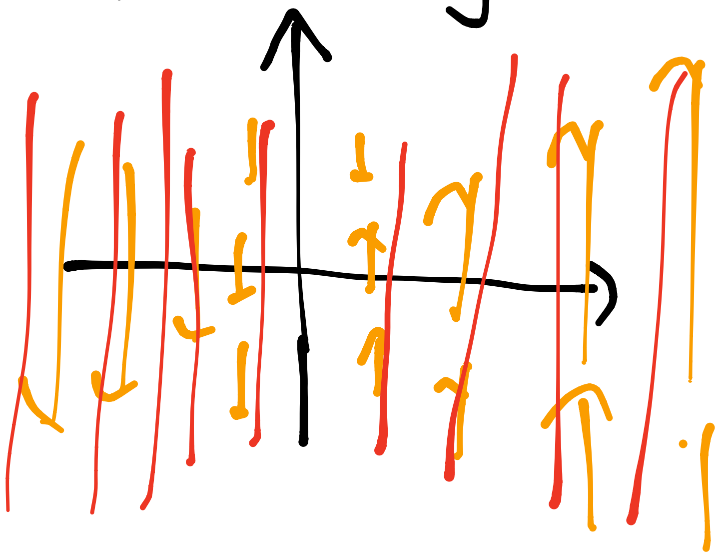
c) $\vec{F}(x,y) = [x,y] = \nabla f$



$f = \frac{x^2}{2} + \frac{y^2}{2}$

d) $\vec{F}(x,y) = [\underset{P}{0}, \underset{Q}{x}]$



$f = ?$

There is none!

$Q_x - P_y = 1$

② $\boxed{\text{Gradient field}}$

$\vec{F}(x,y) = \nabla f(x,y)$

gradient field.

$f(x,y) = x^2 + y^3 x$

$\vec{F}(x,y) = [2x + y^3, 3y^2x]$

How do we get
f from $\vec{F}$?

a) $\vec{F} = [y+x, x+y^7]$
$\qquad\quad f_x \qquad f_y$
What is f?

$$f = xy + \frac{x^2}{2} + \frac{y^8}{8}$$

How do we find it?

b) $\vec{F}(x,y) = [-y+x, x]$
Find f!

~~try~~: $f = -xy + \frac{x^2}{2}$

$F = [-y + x, -x]$

There is no !

How do we see that?

$$F = [P, Q] \quad \text{with } P = f_x,\ Q = f_y$$

$$f_{yx} + f_{xy} = 0$$

$$\boxed{Q_x - P_y = 0}$$

Clairaut ~~test~~

has to be satisfied if $\overline{F} = \nabla f$.

$$\overline{F} = \begin{array}{c} \begin{array}{cc} P & Q \end{array} \\ \left[-y+x, \; x \right] \end{array}$$

$Q_x - P_y = 1 + 1 = 2$

is not zero!

we will call

$Q_x - P_y$ the

curl of $\overline{F}$

③ $\boxed{\text{How do we find } f\text{?}}$

$$\vec{F} = \begin{bmatrix} \sin x + y + 2yx \\ \cos y + \cancel{x} + x^2 + y^7 \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Find f (potential)

Ⓐ Integrate the first equation $P = f_x$ w.r.to x

$$f = -\cos x + xy + x^2 y + C(y)$$

depends on y

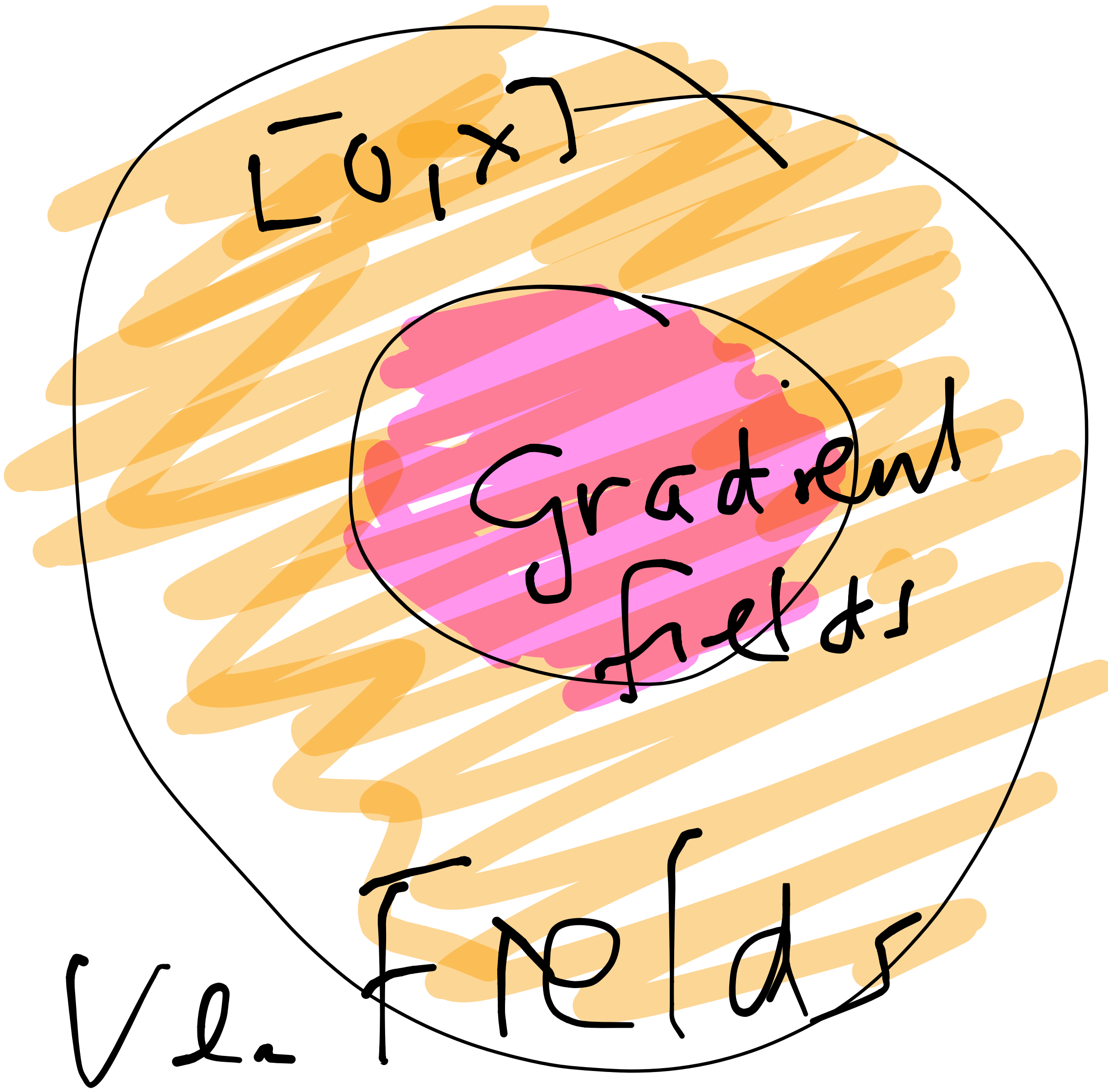
(B) Differentiate f w.r.to y.

$$f_y = \cancel{x} + \cancel{x^2} + C'(y)$$
$$= \cos y + \cancel{x} + \cancel{x^2} + y^7$$

$$C'(y) = \cos y + y^7$$

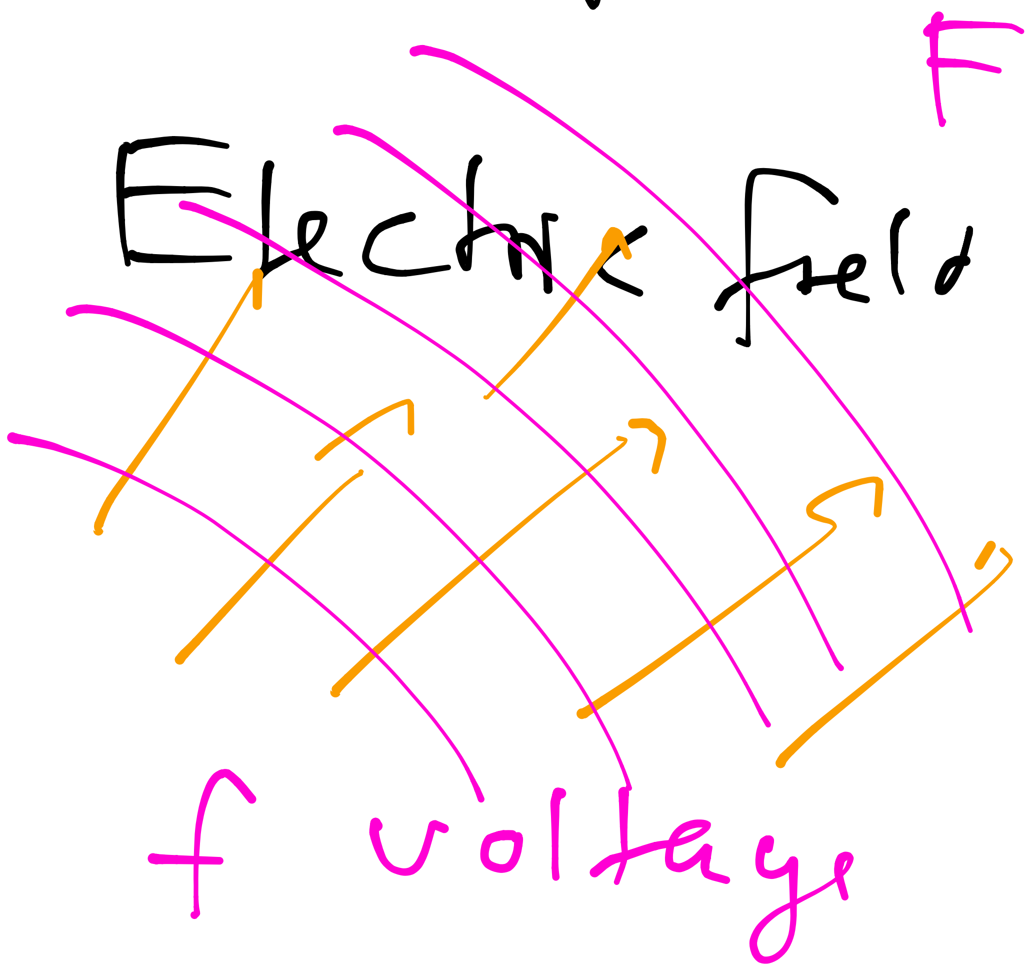
$$C(y) = \sin y + \frac{y^8}{8}$$

$$\boxed{f = -\cos x \cdot 1 \, xy \cdot 1 \, x^2 y + \sin y + \frac{y^8}{8}}$$



Force fields are gradient field

Gravi? field



F
Electric field
f voltage