7/9/2020 FIRST HOURLY **Maths 21a, O.Knill, Summer 2020**

"I affirm my awareness of the standards of the Harvard College Honor Code."

Name:

Please email the PDF as an email attachment to knill@math.harvard.edu. The file needs to have your name capitalized like OliverKnill.pdf. Use **your personal handwriting**, no typing. No books, calculators, computers, or other electronic aids are allowed. (You can use a tablet to write). You can consult with a single page of your own handwritten notes, when writing the exam. The exam needs to arrive on Friday, July 10 at 10 AM. Write clearly and always give details of your computations. If you use separate paper, sign it with the honor code statement, use a page for each problem and copy the structure of the **check boxes**. All your final answers need to be in the boxes.

Problem 1) (20 points) No justifications are needed.

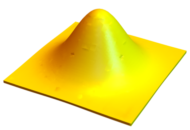
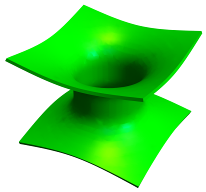
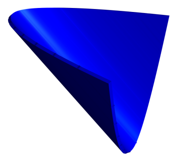
1)	T	F	The plane $x = 3$ does intersect the yz-plane.
2)	T	F	The curve $\vec{r}(t) = [1 + 2t, t, 1 + t]$ intersects the z-axis in a point.
3)	T	F	The Cauchy-Schwartz inequality states $\|\vec{v} \cdot \vec{w}\| \leq \|\vec{v}\|$ for any two vectors $\vec{v}, \vec{w}$.
4)	T	F	The curvature of a circle $\vec{r}(t) = [\cos(2t), 0, \sin(2t)]$ is equal to 1/2 everywhere.
5)	T	F	The surface $y^2 - x + y^2 = 2$ is an elliptic paraboloid.
6)	T	F	The angle between the vectors $\vec{0A}$ and $\vec{0B}$ is positive if the distance between the points A and B is positive.
7)	T	F	Let $\vec{j} = [0, 1, 0]$. There is a vector $\vec{v}$ for which the vector projection of $\vec{v}$ onto $\vec{j}$ is equal to $-\vec{j}$.
8)	T	F	Two particles with path $\vec{r}_1(t) = [0, t, -t]$ and $\vec{r}_2(t) = [1 - t, t - 1, 0]$ do collide.
9)	T	F	In spherical coordinates the surface $\rho^2 \sin^2(\phi) - \rho^2 \cos^2(\phi) = 1$ is a one-sheeted hyperboloid.
10)	T	F	If $\|\vec{u} \times \vec{v}\| = 1$, for unit vectors $\vec{u}, \vec{v}$, then $\vec{u}, \vec{v}$ are orthogonal.
11)	T	F	The curve $r^2 \cos^2(\theta) - r^2 \sin^2(\theta) = 1$ in polar coordinates is a hyperbola.
12)	T	F	If the arc length of a curve connecting A with B is 0, then $A = B$.
13)	T	F	The surface parametrized as $\vec{r}(y, z) = [y, z, y^2 - z^2]$ is a hyperbolic paraboloid.
14)	T	F	The velocity vector and the acceleration are always either parallel or perpendicular.
15)	T	F	It is possible that a plane and a one-sheeted hyperboloid intersects in two crossing lines.
16)	T	F	The function $f(x, y) = \log(x^2 + y^2)$ contains as domain all points except the origin $(0, 0)$.
17)	T	F	The normal vector $\vec{N}$ and the unit tangent vector $\vec{T}$ are perpendicular if $\vec{T}, \vec{T}'$ are both not zero vectors.
18)	T	F	The distance between two non-parallel lines in three dimensional space can be zero.
19)	T	F	If $\vec{i}, \vec{j}, \vec{k}$ denote the unit vectors in the x, y and z axis, then $\vec{i} \cdot (\vec{j} \times \vec{k}) = 1$.
20)	T	F	For any two lines L, M, there are points P on L and Q on M such that $d(P, Q) = 2d(L, M)$, where $d(L, M)$ is the distance between the lines.

Problem 2) (10 points) No justifications are needed in this problem.

In each sub-problem, each of the numbers 0,1,2,3 each occur exactly once.

a) (2 points) Match the surfaces $g(x, y, z) = c$. Enter 0 if there is no match.

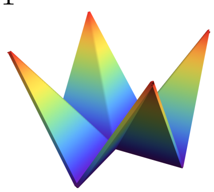
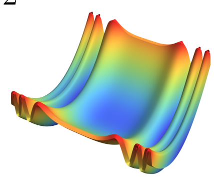
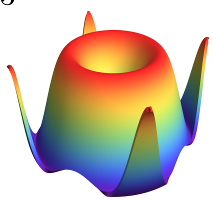
1 2 3

Function $g(x, y, z) =$	0,1,2, or 3
$x - y^2 + z = 0$	
$x^2 + y^2 - z^4 = 1$	
$z - e^{-x^2-y^2} = 0$	
$y^2 + z^3 = 1$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter 0 if there is no match.

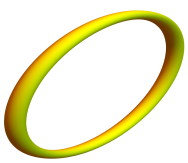
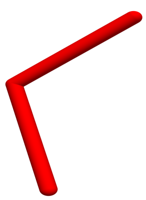
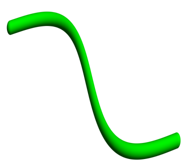
1 2 3

Function $f(x, y) =$	0,1,2, or 3
$\sqrt{\|1 + x^2 - y^2\|}$	
$\sin(x^2 + y^2)$	
$\|\|x - y\| - \|x + y\|\|$	
$y^2 \sin(x^4)$	

c) (2 points) Match the space curves with the parametrizations. Enter 0 if there is no match.

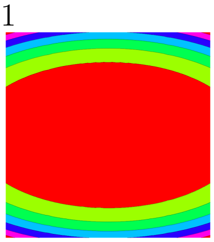
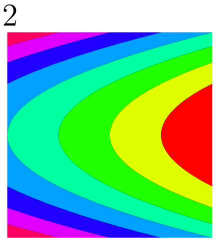
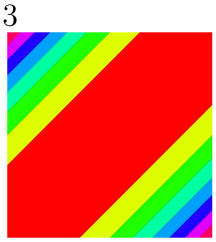
1 2 3

Parametrization $\vec{r}(t) =$	0,1,2, or 3
$[\|t\|, t, t]$	
$[\cos(2t), 0, \cos(2t)]$	
$[0, \cos(2t), \sin(2t)]$	
$[t, \sin(t), 0]$	

d) (2 points) Match the functions g with contour plots in the xy-plane. Enter 0 if there is no match.

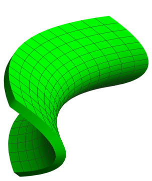
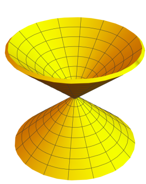
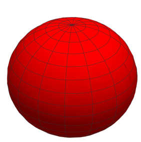
1 2 3

Function $g(x, y) =$	0,1,2, or 3
$\cos(2x) + \sin(2y)$	
$(x - y)^2$	
$y^2 - x$	
$(2x^2 + 7y^2)^2$	

e) (2 points) Match the parametrized surfaces. Enter 0 if there is no match.

1 2 3

Parametrization $\vec{r}(u, v) =$	0-3
$[u, u^2 + v^2, v]$	
$[\sin(v)\cos(u), \sin(v)\sin(u), \cos(v)]$	
$[u^2 - v^2, u, v]$	
$[u\cos(v), u\sin(v), u]$	

Problem 3) (10 points)

Let $\vec{v} = [2, 2, 1]$ and $\vec{w} = [1, 1, 1]$ and assume θ is the angle between $\vec{v}$ and $\vec{w}$.

a) (5 points) Compute $\vec{n} = \vec{v} \times \vec{w}$ and $a = \vec{v} \cdot \vec{w}$.

$\vec{n} =$ | $a =$

b) (5 points) Compute $\sin^2(\theta)$ by using the vector $\vec{n}$ and $\cos^2(\theta)$ by using the scalar a and check that $\sin^2(\theta) + \cos^2(\theta)$ is equal to 1.

$\sin^2(\theta) =$ | $\cos^2(\theta) =$

Check that the sum is 1:

Problem 4) (10 points)

a) (5 points) Parametrize the plane $4x + 4y + 2z = 0$ using parameters s, t:

$\vec{r}(s,t) =$

b) (5 points) Find the distance between $4x + 4y + 2z = 0$ and the point $(2, 2, 2)$.

distance =

Problem 5) (10 points)

a) (5 points) Find a parametrization $\vec{r}(t)$ of the intersection of the planes

$$x + y + z = 1, \qquad 2x - y + 2z = 2 \ .$$

$\vec{r}(t) =$

b) (5 points) Find the distance between that line computed in a) and $P = (1, 1, 1)$.

distance =

Problem 6) (10 points)

a) (5 points) Find the arc length of the path

$$\vec{r}(t) = [\frac{3t^2}{2}, \frac{4t^2}{2}, \frac{5t^3}{3}]$$

with $0 \leq t \leq 1$.

Length =

b) (5 points) Find the curvature of $\vec{r}(t)$ at $t = 1$ using the vectors $\vec{v} = \vec{r}\,'(1), \vec{w} = \vec{r}\,''(1)$.

$\kappa(\vec{r}(1))) =$

Problem 7) (10 points)

a) (5 points) Given the **jerk**

$$\vec{r}\,'''(t) = [0, 0, -12]$$

with $\vec{r}(0) = [0, 0, 3], \vec{r}\,'(0) = [4, 0, 0], \vec{r}\,''(0) = [0, 2, 0]$, find $\vec{r}(t)$ and especially $\vec{r}(10)$.

$\vec{r}(10) =$

b) (5 points) Compute the unit tangent vector $\vec{T}$ of the TNB-frame to the curve $\vec{r}(t)$ at $t = 0$.

$\vec{T}(0) =$

Problem 8) (10 points)

We experiment with a **paper air plane**. The wing tips are $C = (0, 3, 4)$ and $D = (0, -3, 4)$ the front is $A = (3, 0, 0)$ the back is $B = (-3, 0, 0)$.

a) (5 points) The sum of the areas of the triangles ABC and ABD is the total wing area. Find the wing area.

Area =

b) (5 points) The volume of the tetrahedron with vertices A, B, C, D is known to be 1/6th of the volume of the parallelepiped spanned by AB, AC, AD. Find the volume of the tetrahedron.

Volume =

Problem 9) (10 points) No justifications are needed.

a) (2 points) Parametrize the surface $x^2/9 + y^2/4 + z^2/4 = 1$.

$$\vec{r}(\theta, \phi) = [\qquad , \qquad , \qquad]$$

b) (2 points) Parametrize the surface $x^2 = y^2 + z^2$ using an angle θ in the yz-plane.

$$\vec{r}(\theta, x) = [\qquad , \qquad , \qquad]$$

c) (2 points) Parametrize the surface $y = x^4 - 6x^2z^2 + z^4$.

$$\vec{r}(x, z) = [\qquad , \qquad , \qquad]$$

d) (2 points) Parametrize the surface $x^2 + z^2 = 4$ using an angle θ in the xz-plane.

$$\vec{r}(\theta, y) = [\qquad , \qquad , \qquad]$$

e) (2 points) Parametrize the surface $(x-7)^2 + z^2 - y^2 = 8$.

$$\vec{r}(\theta, y) = [\qquad , \qquad , \qquad]$$

The following pictures are here just to be admired and can also be ignored.