

# MULTIVARIABLE CALCULUS

MATH S-21A

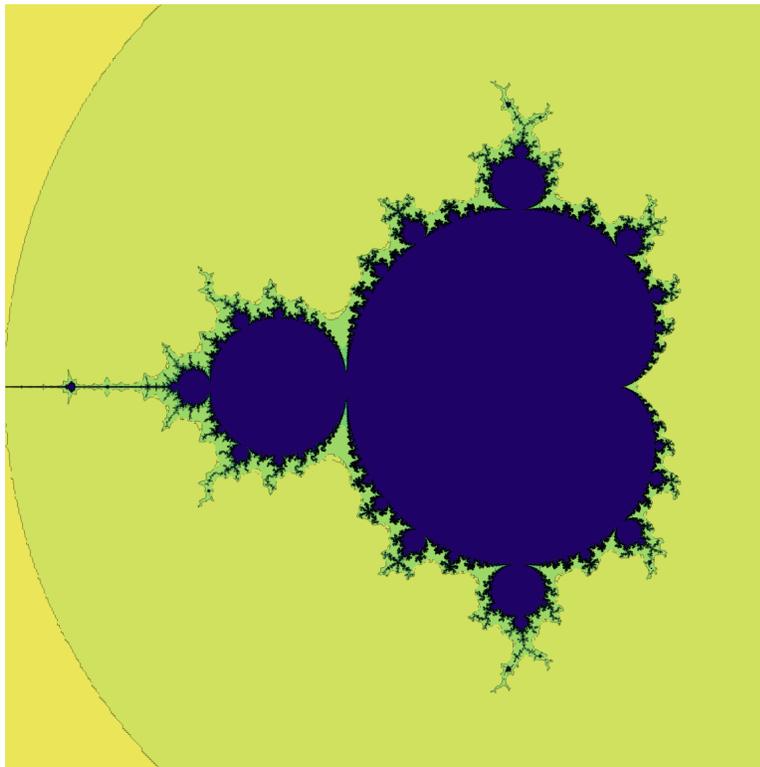
## Data project Week 4: Monte Carlo computation

**2.1.** Often, when we deal with real data, we do not have analytic expressions for the region or function we want to integrate. We want to elaborate here on an example mentioned in the text for unit 15. It is the problem to find the area of Mandelbrot set

$$M = \{c = a + ib \in \mathbb{C} \in \mathbb{R}^2 \mid T_c(0)^n \text{ stays bounded} \},$$

where  $T_c(z) = z^2 + c$  (as complex numbers, which is written out in real coordinates the map  $T_c(x, y) = (x^2 - y^2 + a, 2xy + b)$ ).

**2.2.** Here is a picture: it can also be visualized as a function which is 1 on the Mandelbrot set and 0 else.



**2.3.** What is the area of the Mandelbrot set? We know it is contained in the rectangle  $x \in [-2, 1]$  and  $y \in [-3/2, 3/2]$ . We now just randomly shoot into this rectangle and see whether we are in the Mandelbrot set or not after 1000 iterations. Here is

some Mathematica code which allows you to compute things. When we ran it, it gave a value of about 1.515.... More accurate measurements reported hint for a slightly smaller value like 1.506.... Others have given bounds [1.50311, 1.5613027].

```
M=Compile[{x,y},Module[{z=x+I y,k=0},
  While[Abs[z]<2.&&k<1000,z=N[z^2+x+I y];++k];Floor[k/1000]];
9*Sum[M[-2+3 Random[],-1.5+3 Random[]],{1000000}]/1000000
```

**2.4.** Here are the problems for Friday the 16th in your group. Please make again slides. I know many of you struggle with time. Split the work.

**Problem 1:** Sketch in words in one slide what the Monte Carlo method is. Who is credited for its invention. Why Monte Carlo? What are applications?

**Problem 2:** What is the Mandelbrot set? What is the significance in mathematics, in pop culture?

**Problem 3:** How accurately can you compute the area of the Mandelbrot set with the above code? Which parameters do you need to tune to make it more precise? Experiment.

**Problem 4:** What is the Mandelbulb set? Try to find out about the history.

**Problem 5:** How would you go about setting up a Monte Carlo computation for the volume of the Mandelbulb? This is a bit of a research task as we have not seen a serious estimate for the volume of the Mandelbulb.

(\*) Some reference from 2013 <http://people.math.harvard.edu/~knill/slides/boston/sic.pdf>