Unit 5

① Functions of 2 variables

$$f(x,y) = 2 - x^2 - y^2 = z$$

$$f(1,0) = 1$$

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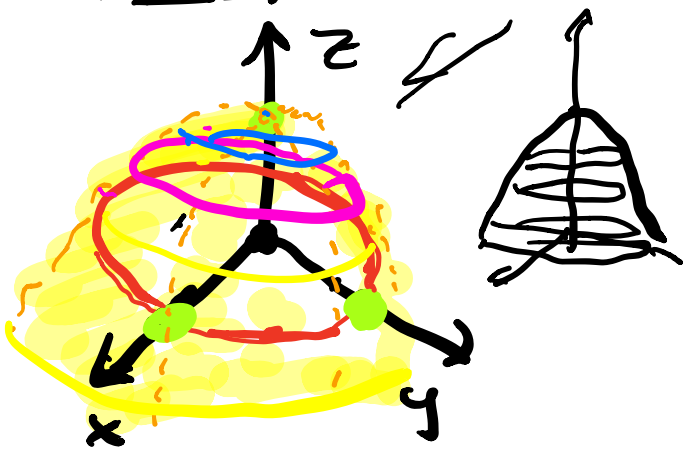
a) Graph

Elliptic Paraboloid

Plot 3D



intercepts

intersections with axes.

traces:

intersection with the coordinate planes

xy-trace : roots

xz-trace $y=0$

$z = 2 - x^2$

$f(x,y) = 0$ **roots**

$2 = x^2 + y^2$

Circle of radius $\sqrt{2}$

xy-trace

generalized traces: $z = 1$

$2 - x^2 - y^2 = 1$

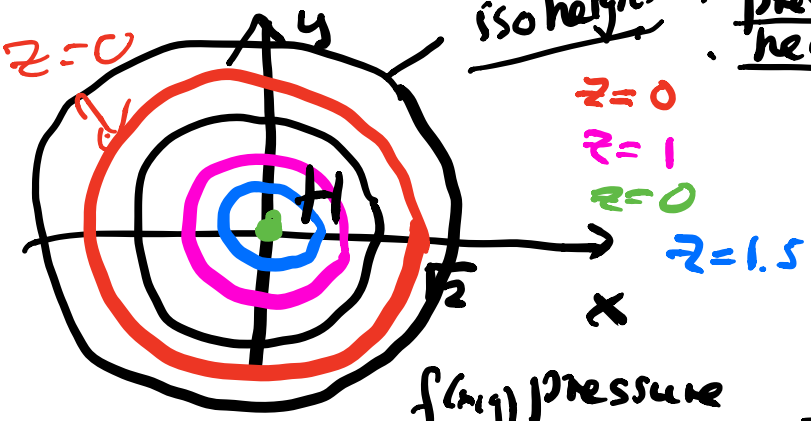
$x^2 + y^2 = 1$

$z = 2$ is just a point

$z = 1.5$

b) **contour map** : temperature, pressure, height

isoheight



$f(x,y)$ pressure — **isobar**

`ContourPlot[f ...]` $f(x,y)$ temperature — **isotherm**

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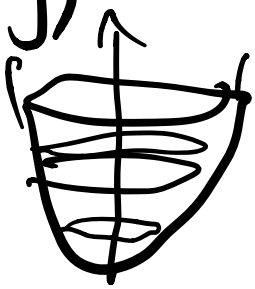
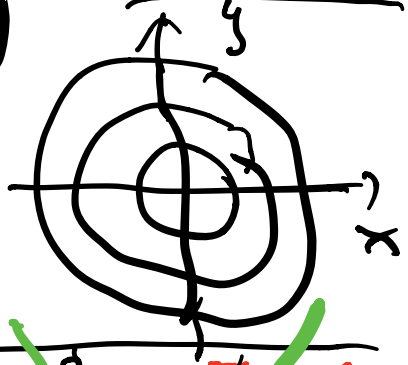
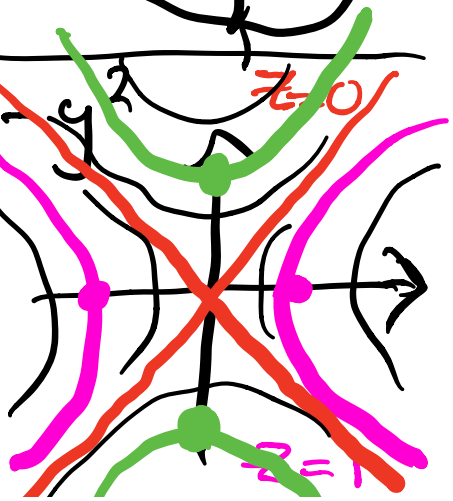
② Examples

(i) $f(x,y) = x^2+y^2$ — Paraboloid





(ii) $f(x,y) = x^2-y^2$

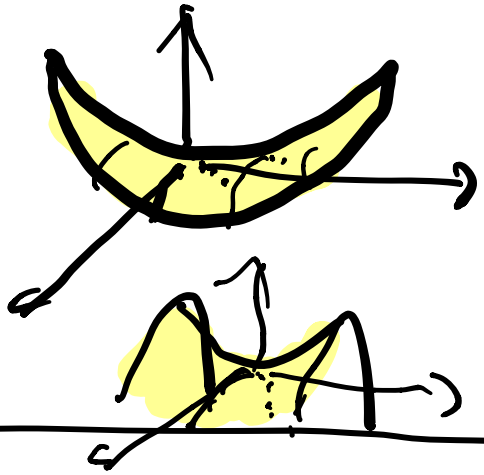


Draw the

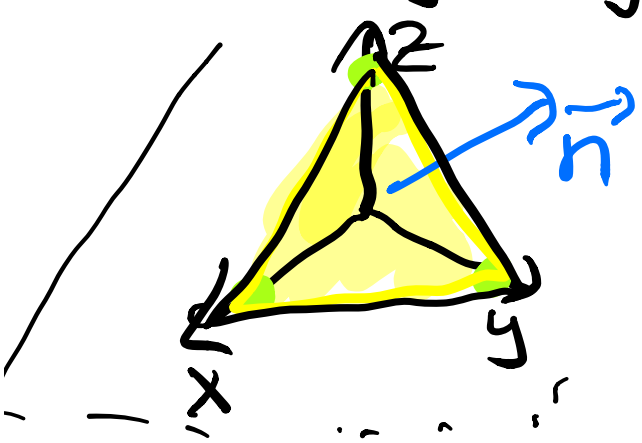
hyperbolic paraboloid

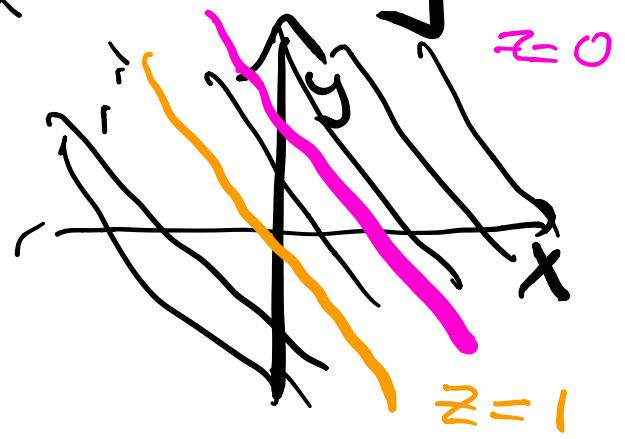
"saddle"

Pringles



(iii) $f(x,y) = 1-x-y = z$





Intercepts $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

$1 = x+y+z$

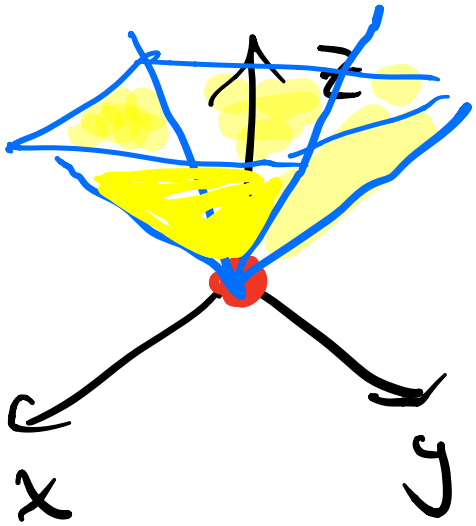
$\boxed{ax+by+cz=d}$

$\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

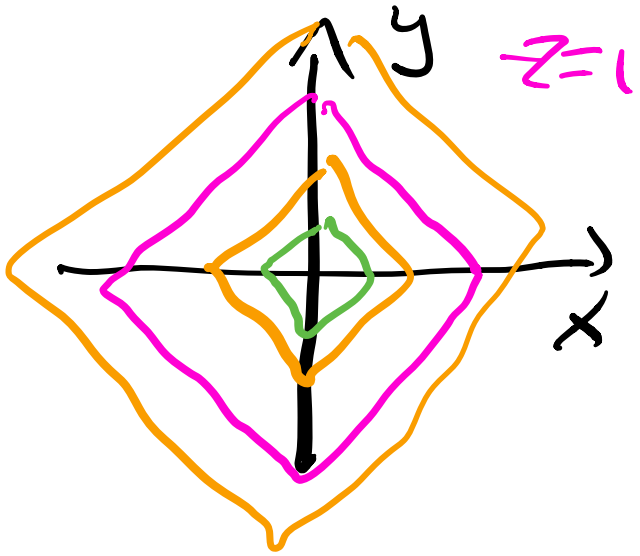
(iv) more difficult case

$f(x,y) = |x| + |y| = z$

distance to the origin in taxi metric.



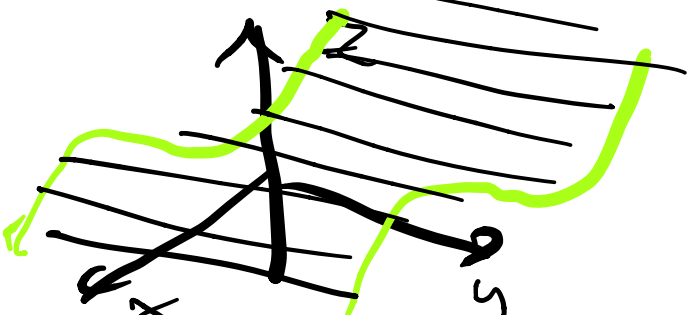
$x = 0$ trace

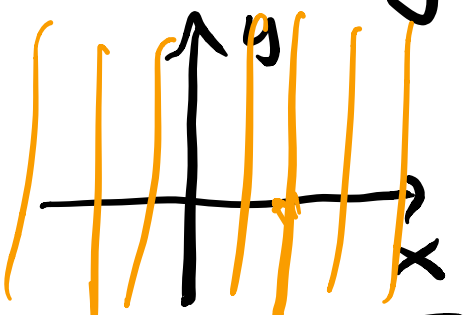


taxi circles

(v) $f(x,y) = x^3 - x = c$

does not involve y.





(3) Functions of 3 variables

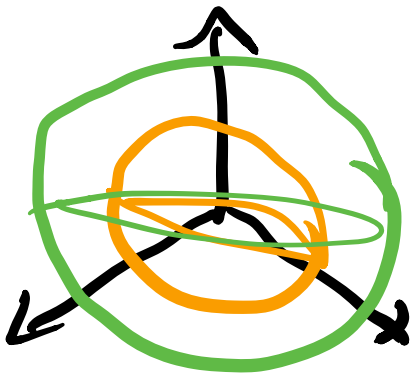
$f(x,y,z)$ can not be visualized as a graph (an object in 4D)

We can look at contours

$f(x,y,z) = c$ = level surface

(i) $f(x,y,z) = x^2 + y^2 + z^2$

The contours are Spheres



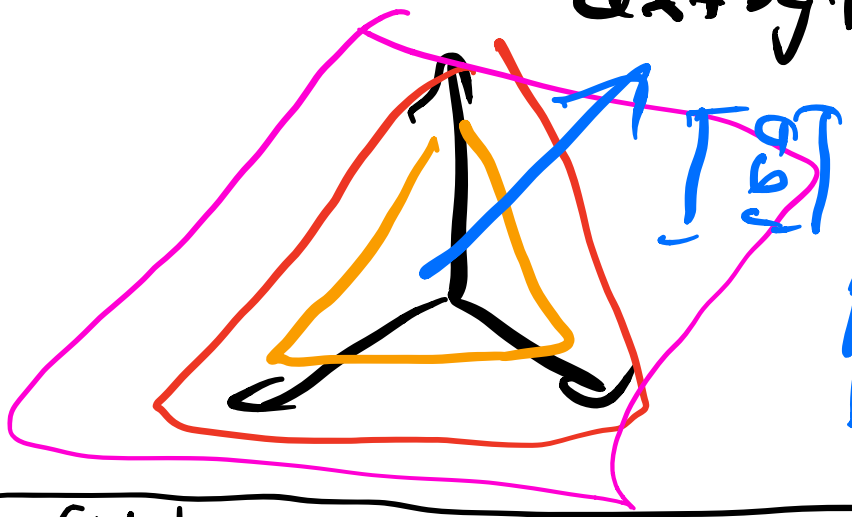
Mathematica

ContourPlot3D

(ii) $f(x,y,z) = ax+by+cz$

The contours are planes

$ax+by+cz = d$



(iii) $g(x,y,z) = f(x,y) - z$

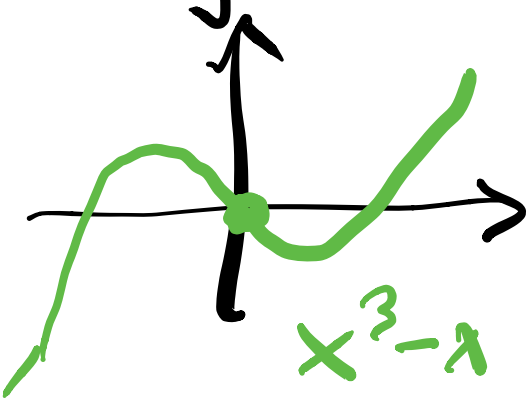
The contour $g(x,y,z)=0$ is the graph of the function $f(x,y)$

This will be important because we will have a theorem.

$f(x,y) = 0$

$f(x,y) = g(x) - y$

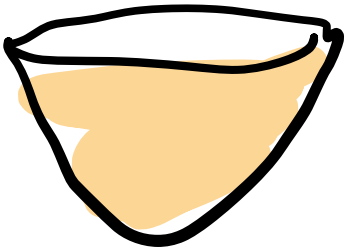
$y = g(x)$



graph of g is a level curve of $f(x,y)$

4 Quadrics

- Paraboloids
  - elliptic paraboloid
  - hyperbolic paraboloid
- Ellipsoids
  - sphere
- Hyperboloids
  - one sheeted
  - two sheeted

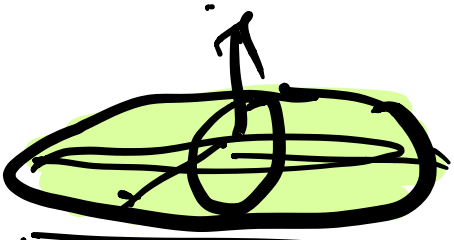


elliptic paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

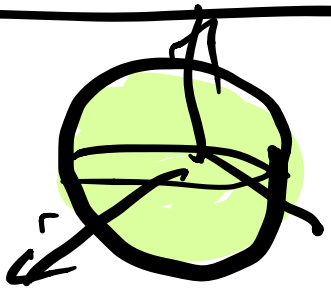
hyperbolic paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$



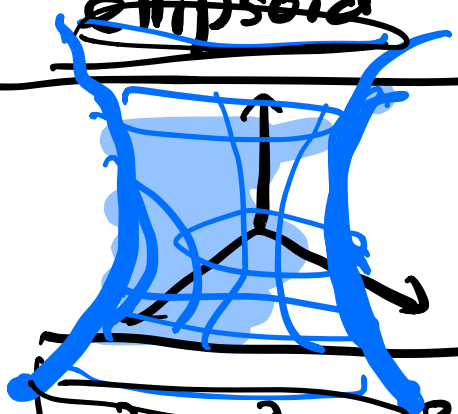
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

ellipsoid



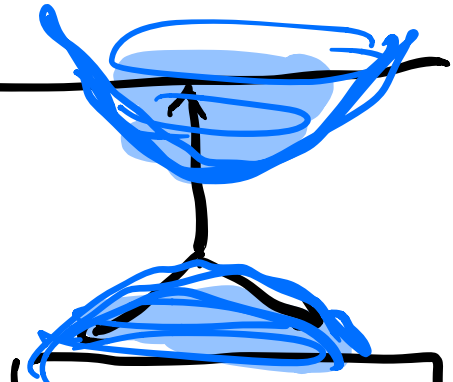
$$x^2 + y^2 + z^2 = 1$$

Sphere



$$x^2 + y^2 - z^2 = 1$$

one sheeted

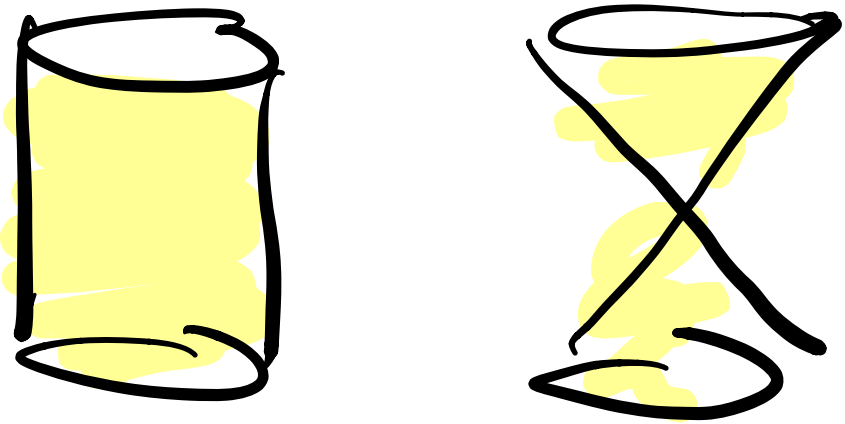


$$x^2 + y^2 - z^2 = -1$$

two sheeted

hyperboloid hyperboloid

Singular cases:



$x^2+y^2=1$ $x^2+y^2-z^2=0$

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(5) **More examples**

See Mathematica demo