Unit 10

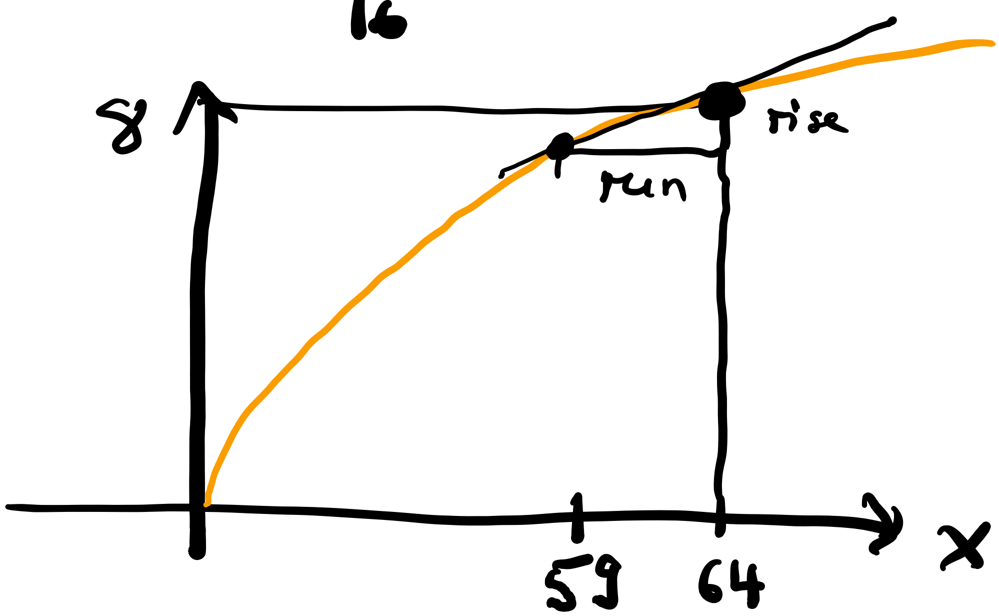
Linearization

1) Magic

(reminders)

$$\sqrt{59} = 7.68115$$

$$8 - \frac{5}{16} = 7.6875$$



$$f(x) = \sqrt{x}, \quad f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(64) = \frac{1}{2 \cdot 8} = \frac{1}{16}$$

$$f'(64) \cdot 5 = \frac{5}{16}$$

$$f(59) \sim 8 - \frac{5}{16}$$

$$\sqrt[3]{999} \simeq ?$$

$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3} x^{-2/3}$$

$$f(1000) = 10$$

$$f'(1000) = \frac{1}{3 \cdot 100} \approx \frac{1}{300}$$

$$10 - \frac{1}{300} \approx 9.9966$$

$$\sqrt{999} = 9.99667$$

$$\boxed{1, 8, 27, 64, 125, 216, 343, 512, 729, 1000}$$

Shakuntala Devi 2019.

"Math in Movies"

This magic is based on Linearization of $f(x)$

$$\boxed{L(x) = f(x_0) + f'(x_0)(x - x_0)}$$

$\longrightarrow$ Taylor expansion.

② Linearization in 2D.

$f(x,y)$ function of 2 variables.

$$\boxed{\begin{aligned} L(x,y) = {} & f(x_0,y_0) \\ & + f_x(x_0,y_0)(x-x_0) \\ & + f_y(x_0,y_0)(y-y_0) \end{aligned}}$$

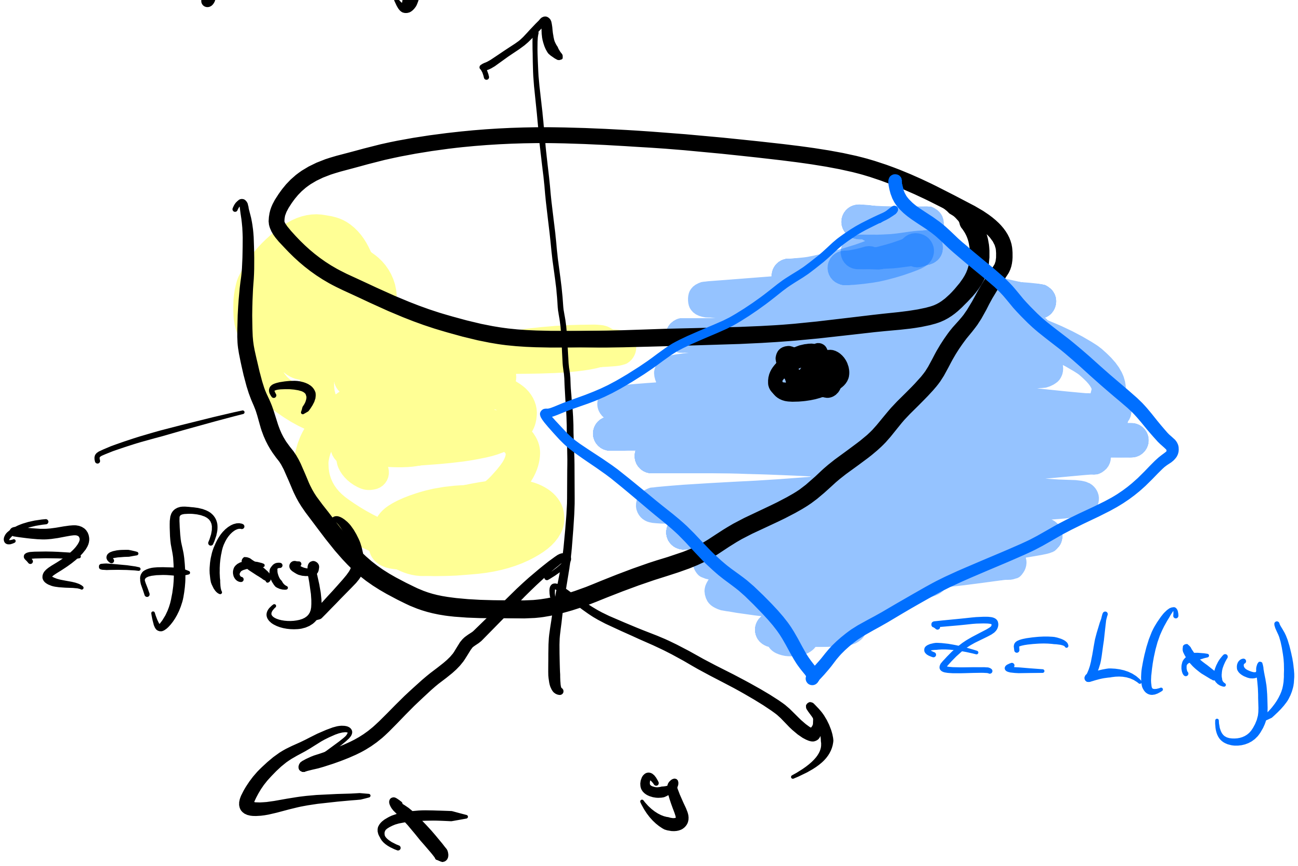
is the linearization of f at (x_0,y_0).

Example: What is the linearization of $f(x,y)=x^4+y^4$ at $(x_0,y_0)=(1,2)$

$$L(x,y) = 17 + 4\cdot 1^3(x-1)$$

$$+ 4\cdot 2^3(y-2)$$

By the way: the graph of $L(x,y)$ is a plane tangent to the graph of f at $(1,2)$



Linearization is very important. Many laws is physics are linearization of more complicated laws.

$$z = \sqrt{1-x^2-y^2} = f(x,y)$$

For the entire sphere

$x^2+y^2+z^2=1 \longrightarrow$ on Thursday

If $f(x,y)$ is a function then

$$\nabla f(x_0, y_0)$$

$$= [f_x(x_0, y_0), f_y(x_0, y_0)]$$

The Linearization can be rewritten as

$$\boxed{f(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0)\cdot(\vec{x}-\vec{x}_0)}$$

$$\nabla f(x_0, y_0)\cdot(x - x_0, y - y_0)$$

$$= f_x(x_0, y_0)(x - x_0)$$

$$+ f_y(x_0, y_0)(y - y_0)$$

In 3 dim things are the same

$$L(x,y,z) = f(x_0,y_0,z_0)$$
$$+ f_x(x_0,y_0,z_0)(x-x_0)$$
$$+ f_y(x_0,y_0,z_0)(y-y_0)$$
$$+ f_z(x_0,y_0,z_0)(z-z_0)$$

$$\boxed{L(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0)\cdot(\vec{x}-\vec{x}_0)}$$

In 3Dim:

$$\boxed{\nabla f(x,y,z) = [f_x, f_y, f_z]}$$

$\longrightarrow$ thursday lecture which features the gradient

∇f

Egyptian hair

"nabla", "gradrew"