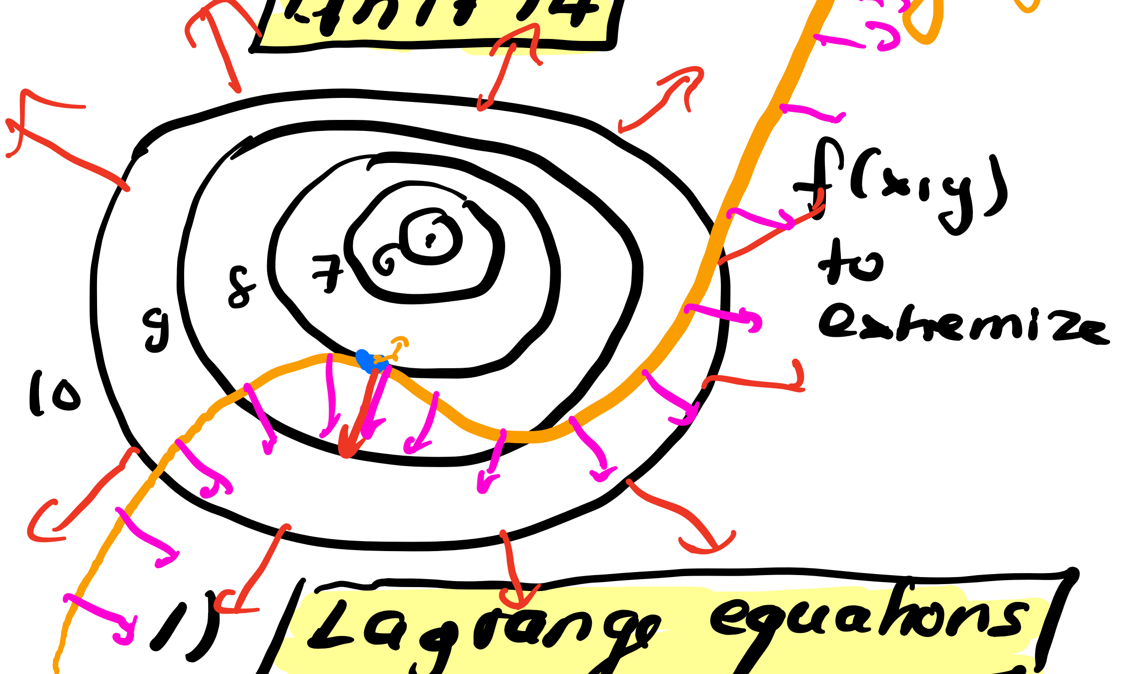
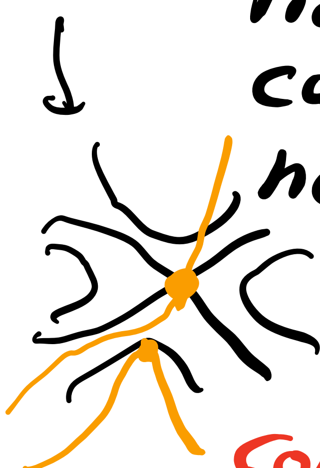
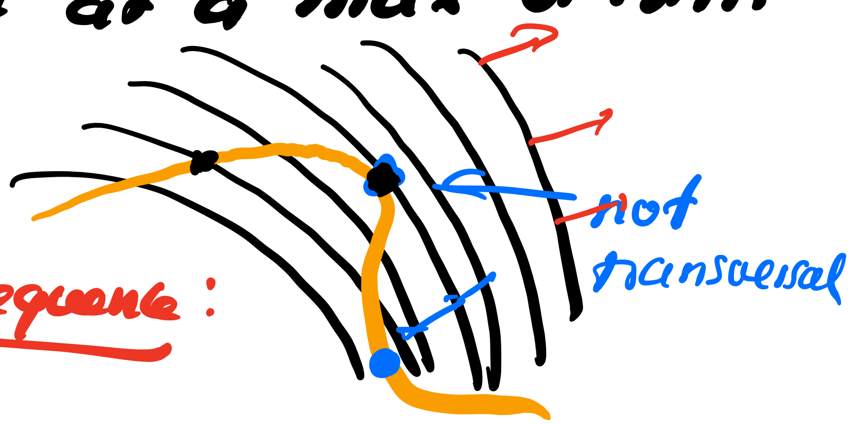
Unit 14



1) Lagrange equations

If the constraint is transversal to a level curve of f, we are not at a max or min





Consequence:

At a max or min ∇f is parallel to ∇g.

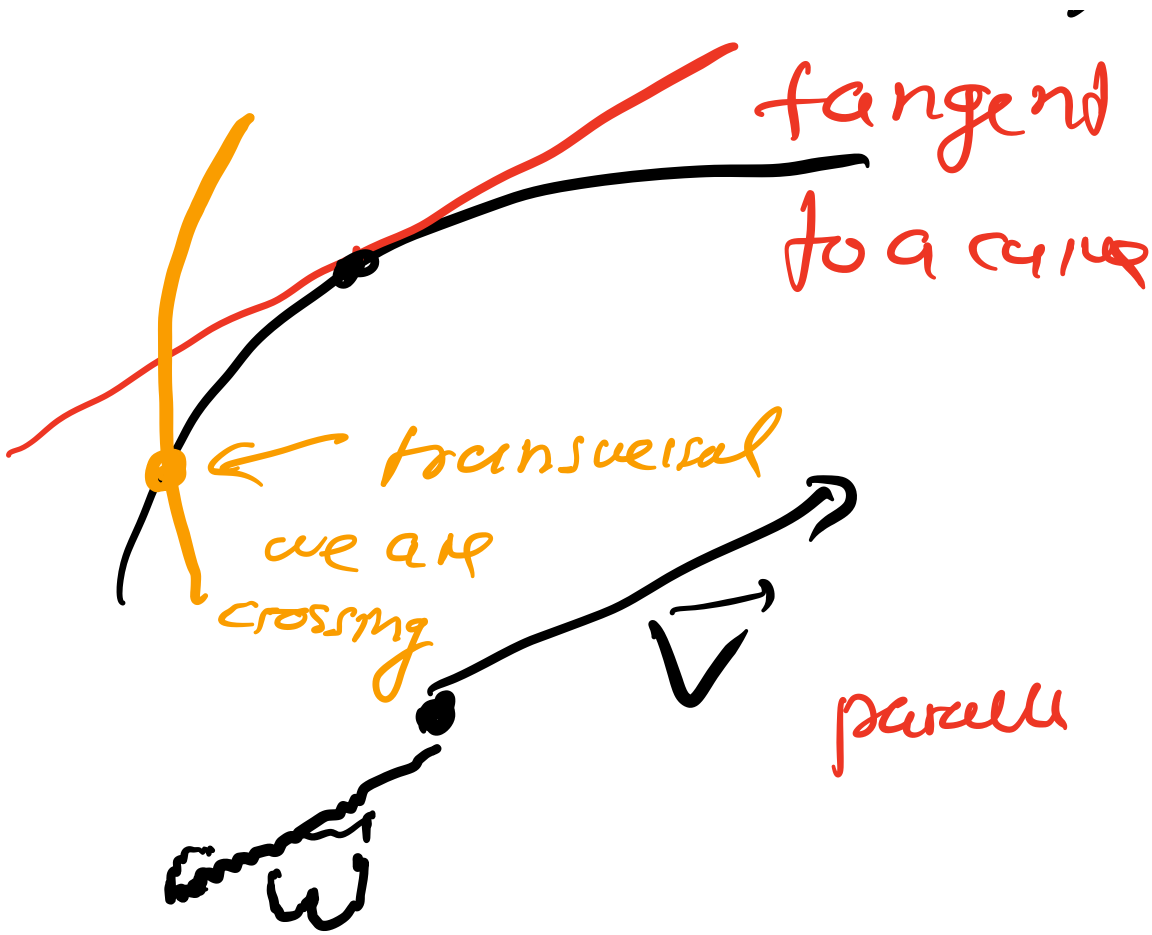
2 equ. $\boxed{\begin{array}{c}\nabla f = \lambda \nabla g \\ g = c\end{array}}$ 1 equ.

λ is an additional parameter

Solutions to this equation give you candidates for maxima and minima.

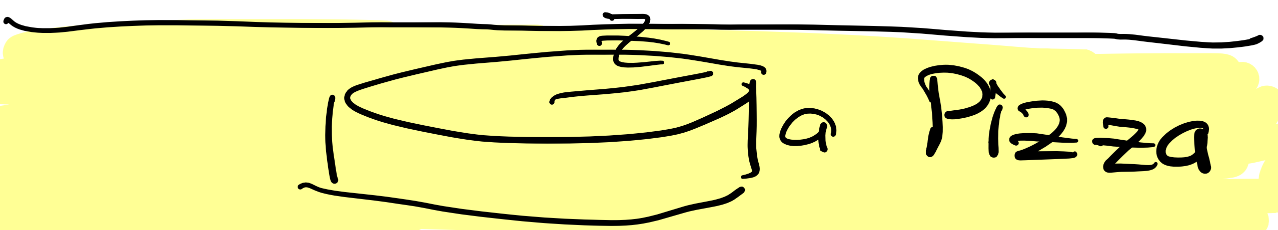
λ = "Lamdaa" is the usal name for the constant. It is honoring Lagrange.

- Usually, we are not interested in λ. Eliminate it first.



$$\bar{v} = \lambda \bar{w}$$

$\bar{v}$ and $\bar{w}$ are called parallel.

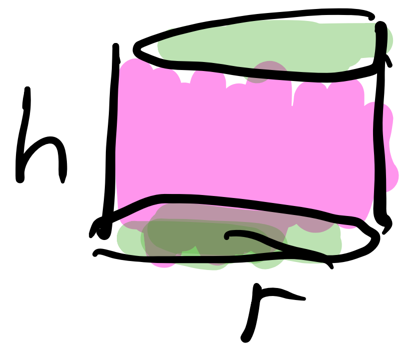


2) $\boxed{\text{Examples}}$ can

$$V = \pi r^2 h$$

$$A = 2\cancel{\pi} r h$$

$$2\cancel{\pi} r^2 = \cancel{\pi}$$



$$f = V = \pi r^2 h \quad \text{Extremize}$$

$$g = A = 2rh + 2r^2 = 1$$

Lagrange equations

$$
\begin{aligned}
&(1) \quad 2\pi r h = \lambda(2h + 4r) \\
&(2) \quad \pi r^2 = \lambda 2r \\
&(3) \quad 2rh + 2r^2 = 1
\end{aligned}
$$

reminder: of Lagrange equations.

here $x = r, \; y = h$

$$\boxed{\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x,y) &= c \end{aligned}}$$

General advise:

get rid of λ immediatly by dividing out and cross multip.

$\frac{(1)}{(2)}$

$$\frac{2\pi r h}{\pi r^2} = \frac{2h + 4r}{2r}$$

$$4\cancel{\pi}\cancel{r^2}h = 2\cancel{r^2}h\cancel{\pi} + 4r^3\cancel{\pi}$$

$r \neq 0 \longrightarrow$

$$2rh + 2r^2 = 1$$

$$\left|\begin{array}{l} \overbrace{4h - 2h}^{2h} = 4r \\ 2rh + 2r^2 = 1 \end{array}\right|$$

$$\left|\begin{array}{l} h = 2r \\ 2rh + 2r^2 = 1 \end{array}\right|$$

plug in: $4r^2 + 2r^2 = 1$

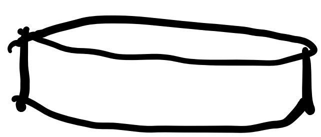
$6r^2 = 1$, $\boxed{r = \frac{1}{\sqrt{6}}}$

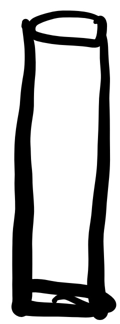
$\boxed{h = \frac{2}{\sqrt{6}}}$

How would we know whether it is is a max or min.

<u>just take any other point.</u>

h small





r

But usually, especially in exams, we don't ask you to do that,

$$f_x = \lambda\ g_x$$
$$f_y = \lambda\ g_y$$

$$f_x \cdot \cancel{\lambda} \cdot g_y = f_y \cdot \cancel{\lambda} \cdot g_x$$

what happens if $\lambda = 0$

then $f_x = 0$, $f_y = 0$.

<u>Mathematically:</u>

$$\nabla f = \lambda \nabla g$$

$$\Longleftrightarrow \quad \nabla f \times \nabla g = 0$$

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} \times \begin{bmatrix} g_x \\ g_y \end{bmatrix} = f_x g_y - g_x f_y$$

③ Entropy

You have a dice with 6 sides.
The probability to hit side k is p_k.

Define the entropy S as $S(p_1, \ldots, p_6)$

$$-p_1 \log p_1 - p_2 \log p_2 - p_3 \log p_3$$
$$- p_4 \log p_4 - p_5 \log p_5 - p_6 \log p_6$$

$$\boxed{S = \sum_{k=1} - p_k \log p_k}$$

Shannon entropy

Bolzman entropy.

Nature likes to maximize entropy!

Entropy measures "disorder".

There is a constraint!

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$$

$$\frac{d}{dp_1} S = \lambda\, 1$$

$$\frac{d}{dp_2} S = \lambda \cdot 1$$

$$\vdots$$

$$\frac{d}{dp_6} S = \lambda\, 1$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$$

Calculus intermezzo

$$f(x) = -x \log x$$

$$f'(x) = -\log x - 1$$

$$-\log p_1 - 1 = \lambda$$

$$-\log p_2 - 1 = \lambda$$

$$\vdots$$

$$-\log p_6 - 1 = \lambda$$

$$p_1 + \dots + p_6 = 1$$

all p_k are the same

So:

$$p_1 = \frac{1}{6}$$

$$p_2 = \frac{1}{6}$$

$$\vdots$$

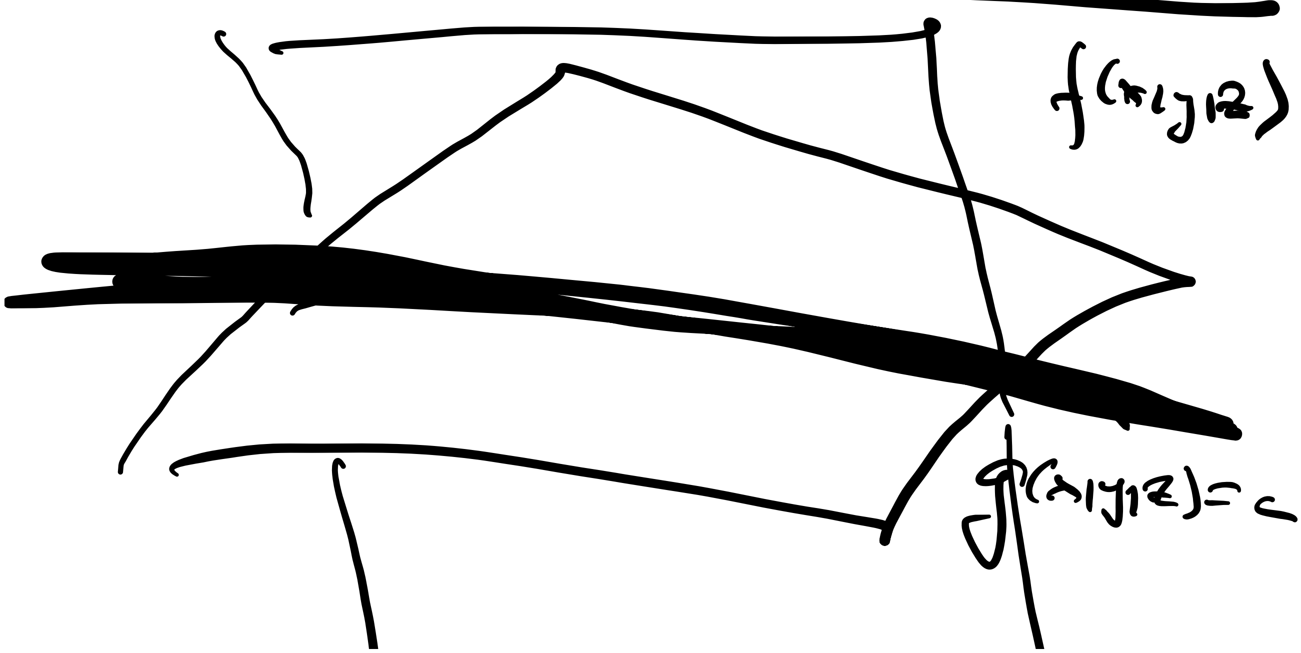
$$p_6 = \frac{1}{6}$$

In the homework, you look

$$S - E = F$$

entropy energy free energy

→ Gibbs distribution.



$h(x,y,z)=d$

$$\boxed{\begin{gathered}\nabla f = \lambda \nabla g + \mu \nabla h \\ g = c \\ h = d\end{gathered}}$$

μ = "μ" is another variable.