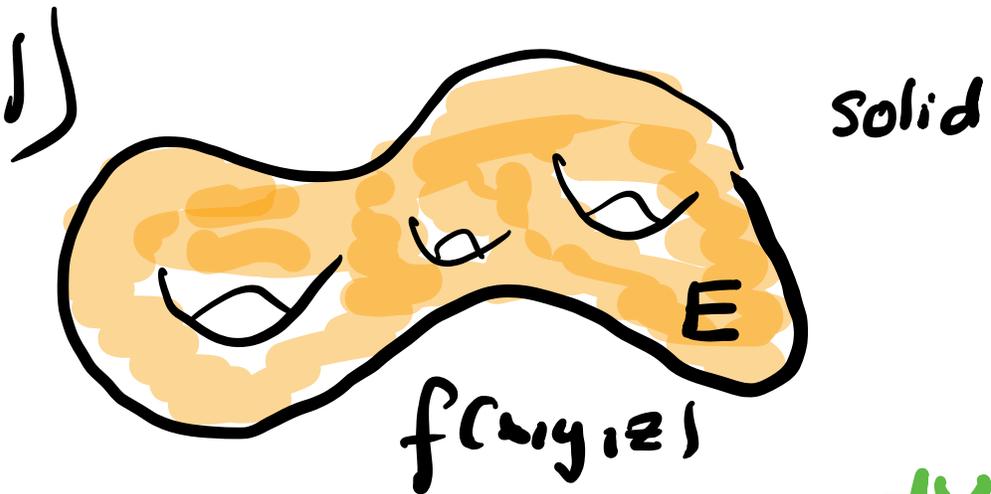


Unit 17

Triple integrals



$$\iiint_E f(x,y,z) \, dx \, dy \, dz$$

dV

↑ mass density

Mass

Important case:

$$f(x,y,z) = 1$$

$$\iiint_E 1 \, dV = \text{Volume}$$

Remindv:  $\int_a^b 1 dx$  Length

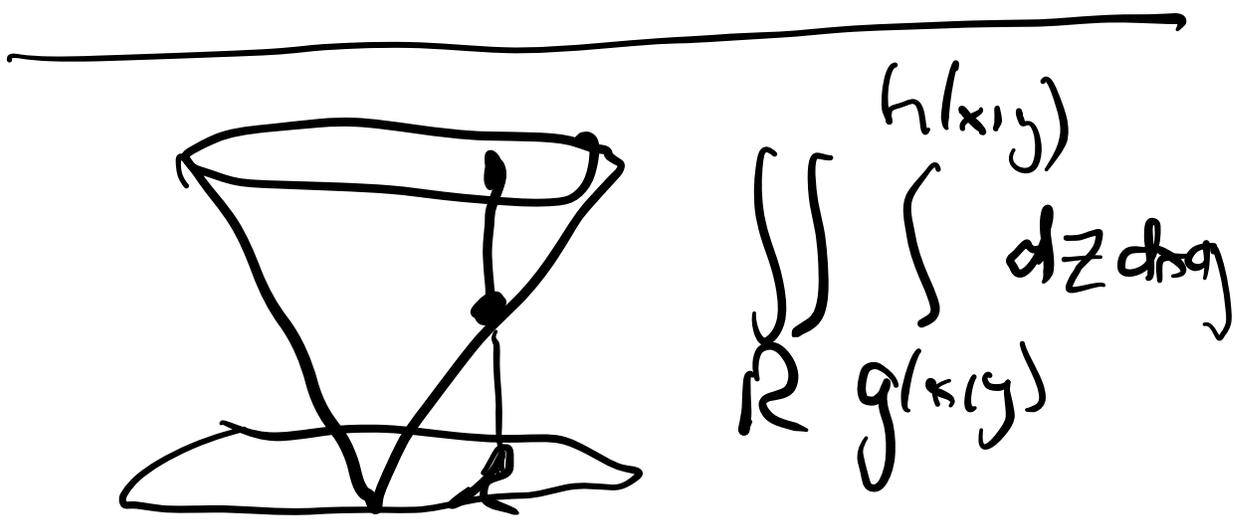
$\iiint_E 1 dV$   
E Volume

$\iint_R 1 dA$  Area

$$\iint_R \int_0^{f(x,y)} T dz \, dx dy$$

$\iint_R f(x,y) \, dx dy$   
Volume under the  
graph of  $f(x,y)$ .





we reduce things  
 to 2D integrals  
 unlike in Math 1

## 2) Typical problem

$$\int_0^{\pi} \int_{\sqrt{z}}^{\sqrt{\pi}} \int_0^x \sin(xy) dy dx dz$$

example of a triple integral. Reduce to a double integr.

Start with integration from the inside. No Fubini!

$$\int_0^{\pi} \int_{\sqrt{z}}^{\sqrt{\pi}} \left. \frac{-\cos(xy)}{x} \right|_{y=0}^{y=x} dx dz$$

$$\int_0^{\pi} \int_{\sqrt{z}}^{\sqrt{\pi}} \frac{1 - \cos(x^2)}{x} dx dz$$

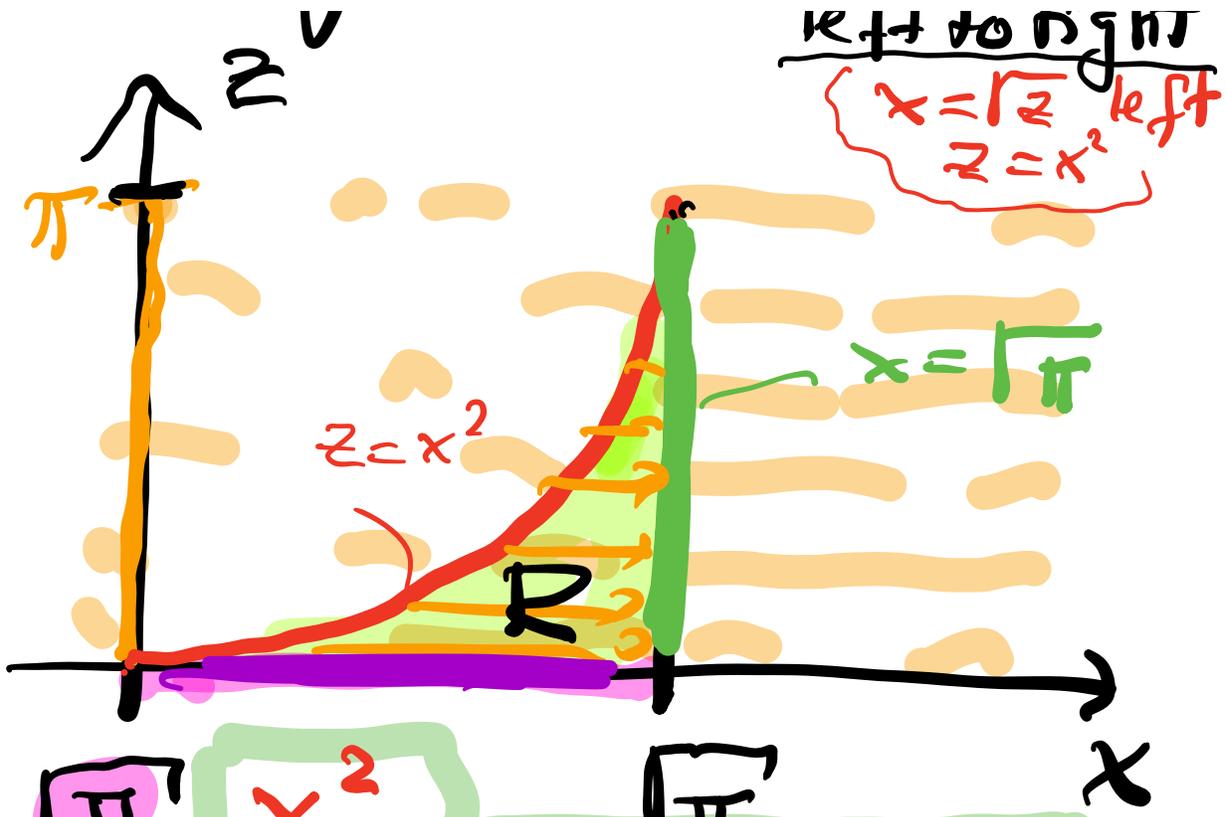
Now we deal with  
 a double integral only  
 This is now review  
 for the exam!

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We can not integrate  
 $\frac{1 - \cos(x^2)}{x}$  !

What do we do?

Change the order of  
 integration,



$$\int_0^{\sqrt{\pi}} \int_0^{x^2} \frac{1 - \cos(x^2)}{x} dz dx$$

$$\int_0^{\sqrt{\pi}} \left( \frac{1 - \cos x^2}{x} \right) (x^2 - 0) dx$$

$$\int_0^{\sqrt{\pi}} x - x \cos x^2 dx$$

$$\left. \frac{x^2}{2} - \frac{\sin(x^2)}{2} \right|_0^{\sqrt{\pi}}$$

$$\boxed{\frac{\pi}{2}}$$

This is a prototype problem.

always constants

$$\int_a^b \int_{g(x)}^{h(x)} f(x,y) dy dx$$

bottom to top  
integral

we can use  
x here

