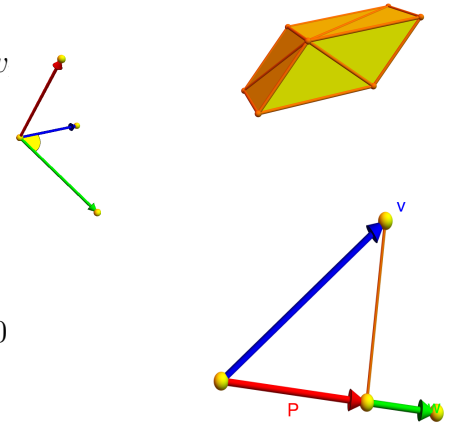


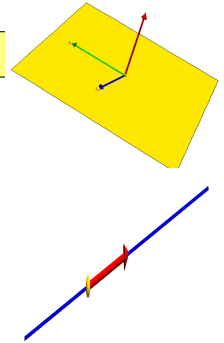
Geometry of Space

- ☐ point $P = (a, b, c)$ with **coordinates** a, b, c .
- ☐ vector $\vec{v} = \vec{PQ} = [v_1, v_2, v_3]$ with components v_k .
- ☐ unit vector = direction = vector of length 1.
- ☐ $\vec{v} = [v_1, v_2, v_3], \vec{w} = [w_1, w_2, w_3], \vec{v} + \vec{w} = [v_1 + w_1, v_2 + w_2, v_3 + w_3]$
- ☐ dot product $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3 = |\vec{v}| |\vec{w}| \cos(\alpha)$
- ☐ angle $\cos(\alpha) = (\vec{v} \cdot \vec{w}) / (|\vec{v}| |\vec{w}|)$
- ☐ cross product $\vec{v} \cdot (\vec{v} \times \vec{w}) = 0, \vec{w} \cdot (\vec{v} \times \vec{w}) = 0$
- ☐ triple scalar product $\vec{u} \cdot (\vec{v} \times \vec{w})$
- ☐ area of parallelogram $|\vec{v} \times \vec{w}| = |\vec{v}| |\vec{w}| \sin(\alpha)$
- ☐ volume of parallelepiped: $|\vec{u} \cdot (\vec{v} \times \vec{w})|$
- ☐ parallel vectors $\vec{v} \times \vec{w} = 0$, orthogonal vectors: $\vec{v} \cdot \vec{w} = 0$
- ☐ scalar projection $\vec{v} \cdot \vec{w} / |\vec{w}|$
- ☐ vector projection $P_{\vec{w}}(\vec{v}) = (\vec{v} \cdot \vec{w}) \vec{w} / |\vec{w}|^2$
- ☐ completion of square: $x^2 - 4x + y^2 = 1$ gives $(x - 2)^2 + y^2 = 5$
- ☐ sphere: $(x - 1)^2 + (y - b)^2 + (z - c)^2 = \rho^2$



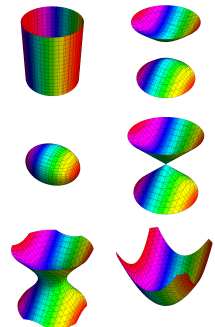
Lines, Planes, Functions

- ☐ parametric equation for plane $\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}$
- ☐ plane $ax + by + cz = d$
- ☐ parametric equation for line $\vec{x} = \vec{x}_0 + t\vec{v}$
- ☐ symmetric equation of line $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
- ☐ graph $G = \{(x, y, f(x, y)) \mid (x, y) \text{ in the domain of } f\}$
- ☐ plane $ax + by + cz = d$ has normal vector $\vec{n} = [a, b, c]$
- ☐ plane through A, B, C : us $[a, b, c] = \vec{AB} \times \vec{CB}$ and plug in point



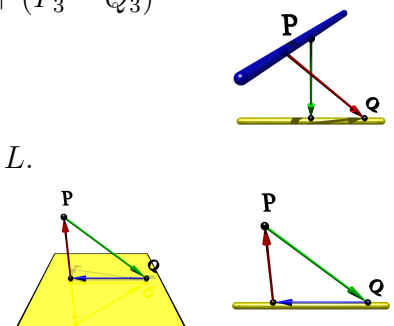
Level surfaces

- ☐ intercepts: intersections of a surface with coordinate axis
- ☐ traces: intersections of a surface with coordinate planes
- ☐ generalized traces: intersections with $\{x = c\}$, $\{y = c\}$ or $\{z = c\}$
- ☐ level surface $g(x, y, z) = c$. Example: graph $g(x, y, z) = z - f(x, y)$
- ☐ linear equation like $2x + 3y + 5z = 7$ defines plane
- ☐ quadric: ellipsoid, paraboloid, hyperboloid, cylinder, cone



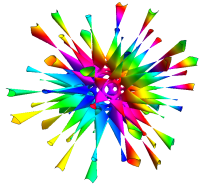
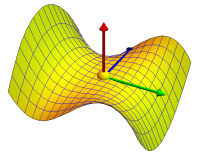
Distance formulas

- ☐ point-point: $d(P, Q) = |\vec{PQ}| = \sqrt{(P_1 - Q_1)^2 + (P_2 - Q_2)^2 + (P_3 - Q_3)^2}$
- ☐ point-plane: $d(P, \Sigma) = |(\vec{PQ}) \cdot \vec{n}| / |\vec{n}|$
- ☐ point-line: $d(P, L) = |(\vec{PQ}) \times \vec{u}| / |\vec{u}|$
- ☐ line-line: $d(L, M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})| / |\vec{u} \times \vec{v}|$
- ☐ parallel lines: L, M : distance point $d(P, M)$, where P is in L .
- ☐ parallel planes: $d(\Pi, \Sigma) = d(P, \Sigma)$, where $P \in \Pi$ is point.



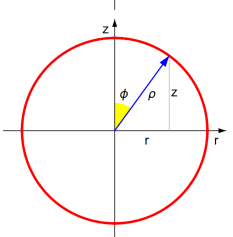
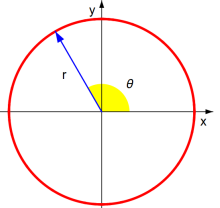
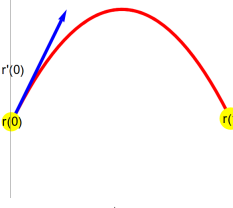
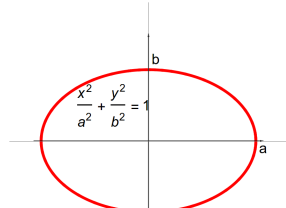
Functions

- ☐ graph: $z = f(x, y)$ is a surface in $\mathbb{R}^3$.
- ☐ contour set: $f(x, y) = c$ is usually a curve in $\mathbb{R}^2$
- ☐ contour map: draw contour sets $\{f(x, y) = c\}$ for various c .
- ☐ contour surface: $f(x, y, z) = c$ is usually a surface in space
- ☐ space contour map: draw contour sets $\{f(x, y, c) = c\}$ for various c .



Curves

- ☐ plane and space curve $\vec{r}(t) = [x(t), y(t)]$, $\vec{r}(t) = [x(t), y(t), z(t)]$
- ☐ circle: $x^2 + y^2 = r^2$, $\vec{r}(t) = [r \cos t, r \sin t]$.
- ☐ ellipse: $(x - x_0)^2/a^2 + (y - y_0)^2/b^2 = 1$, $\vec{r}(t) = [x_0 + a \cos t, y_0 + b \sin t]$
- ☐ velocity $\vec{r}'(t)$, acceleration $\vec{r}''(t)$, $|\vec{r}'(t)|$ speed
- ☐ unit tangent vector $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$
- ☐ integration: get $\vec{r}(t)$ from $\vec{r}'(t)$ and $\vec{r}(0)$ by integration.
- ☐ integration: get $\vec{r}(t)$ from acceleration $\vec{r}''(t)$ as well as $\vec{r}'(0)$ and $\vec{r}(0)$.
- ☐ velocity $\vec{r}'(t)$ tangent to curve at $\vec{r}(t)$.
- ☐ polar curve $\vec{r}(t) = [f(t) \cos(t), f(t) \sin(t)]$ to polar graph $r = f(\theta)$.
- ☐ arc length $\int_a^b |\vec{r}'(t)| dt$ of parametrized curve. Param. independent!
- ☐ normal vector $\vec{N}(t) = \vec{T}'(t)/|\vec{T}'(t)|$ perpendicular to $\vec{T}(t)$.
- ☐ bi-normal $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$ perpendicular to $\vec{T}$ and $\vec{N}$.
- ☐ curvature $\kappa(t) = |\vec{T}'(t)|/|\vec{r}'(t)| = |\vec{r}'(t) \times \vec{r}''(t)|/|\vec{r}'(t)|^3$. Parameter inde
- ☐ torsion $\tau(t) = |\vec{r}' \cdot \vec{r}'' \times \vec{r}'''|/|\vec{r}' \times \vec{r}''|$



Coordinates

- ☐ Cartesian coordinates (x, y, z)
- ☐ polar coordinates $(x, y) = (r \cos(\theta), r \sin(\theta))$, $r \geq 0$
- ☐ cylindrical coordinates $(x, y, z) = (r \cos(\theta), r \sin(\theta), z)$, $r \geq 0$
- ☐ spherical coordinates $(x, y, z) = (\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi))$
- ☐ radius: $r = x^2 + y^2$ and spherical radius $\rho = x^2 + y^2 + z^2 \geq 0$
- ☐ polar radius: important relation $r = \rho \sin(\phi)$

Surfaces

- ☐ $\vec{r}(u, v) = [x(u, v), y(u, v), z(u, v)]$ parametrization
- ☐ $\vec{r}(t) = \vec{r}(u_0, t)$, $\vec{r}(t) = \vec{r}(t, v_0)$ grid curves on surface
- ☐ $g(r, \theta) = 0$ polar curve, especially $r = f(\theta)$, polar graph
- ☐ $r = f(z, \theta)$ cylindrical surface, $r = r(z)$ surface of revolution
- ☐ $g(\rho, \theta, \phi) = 0$ spherical surface: example $\rho = 1$ sphere
- ☐ plane: $ax + by + cz = d$, $\vec{r}(s, t) = \vec{r}_0 + s\vec{v} + t\vec{w}$, $[a, b, c] = \vec{v} \times \vec{w}$
- ☐ surface of revolution: $x^2 + y^2 = r(z)^2$, $\vec{r}(\theta, z) = [r(z) \cos(\theta), r(z) \sin(\theta), z]$
- ☐ graph: $g(x, y, z) = z - f(x, y) = 0$, $\vec{r}(x, y) = [x, y, f(x, y)]$
- ☐ rotated graph $g(x, y, z) = y - f(x, z) = 0$, $\vec{r}(x, z) = [x, f(x, z), z]$
- ☐ ellipsoid: $\vec{r}(\theta, \phi) = [a \cos \theta \sin \phi, b \sin \theta \sin \phi, c \cos \phi]$
- ☐ unit sphere: $x^2 + y^2 + z^2 = 1$, $\vec{r}(u, v) = [\cos u \sin v, \sin u \sin v, \cos v]$

